LECTURE 5:
Bosons and Fermions

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Outline

Lagrangian field theory (I)

Formulate field theory in terms of first order Lagrangian density $\Lambda(\varphi, \partial\varphi, x)$. For a spacetime region M the **action** of a field ϕ is

$$S_M(\phi) := \int_M \Lambda(\phi(\cdot), \partial\phi(\cdot), \cdot).$$

Classical solutions in M are extremal points of this action. These are obtained by setting to zero the first variation of the action,

$$(\mathrm{d}S_M)_\phi(X) = \int_M X^a \left(\frac{\delta\Lambda}{\delta\varphi^a} - \partial_\mu \frac{\delta\Lambda}{\delta\,\partial_\mu\varphi^a}\right)(\phi) + \int_{\partial M} X^a \partial_\mu \lrcorner \frac{\delta\Lambda}{\delta\partial_\mu\varphi^a}(\phi)$$

under the condition that the infinitesimal field X vanishes on ∂M. This yields the **Euler-Lagrange equations**,

$$\left(\frac{\delta\Lambda}{\delta\varphi^a} - \partial_\mu \frac{\delta\Lambda}{\delta\,\partial_\mu\varphi^a}\right)(\phi) = 0.$$

Lagrangian field theory (II)

The boundary term can be defined for an arbitrary hypersurface Σ.

$$(\theta_\Sigma)_\phi(X) = -\int_\Sigma X^a \partial_\mu \lrcorner \frac{\delta\Lambda}{\delta\partial_\mu\varphi^a}(\phi)$$

This 1-form is called the **symplectic potential**. Its exterior derivative is the **symplectic 2-form**,

$$(\omega_\Sigma)_\phi(X,Y) = (\mathrm{d}\theta_\Sigma)_\phi(X,Y) = -\frac{1}{2}\int_\Sigma \Bigg((X^b Y^a - Y^b X^a)\,\partial_\mu \lrcorner \frac{\delta^2\Lambda}{\delta\varphi^b \delta\,\partial_\mu\varphi^a}(\phi) + (Y^a \partial_\nu X^b - X^a \partial_\nu Y^b)\,\partial_\mu \lrcorner \frac{\delta^2\Lambda}{\delta\,\partial_\nu\varphi^b \delta\,\partial_\mu\varphi^a}(\phi) \Bigg).$$

We denote the space of solutions in M by L_M and the space of germs of solutions on a hypersurface Σ by L_Σ.

Lagrangian field theory (III)

Let M be a region and $\phi \in L_{\partial M}$. Then ϕ may or may not be induced from a solution in M. If ϕ arises from a solution in M and X, Y arise from infinitesimal solutions in M, then,

$$(\omega_{\partial M})_\phi(X, Y) = (\mathrm{d}\theta_{\partial M})_\phi(X, Y) = -(\mathrm{d}\mathrm{d}S_M)_\phi(X, Y) = 0.$$

This means, L_M induces an **isotropic** submanifold of $L_{\partial M}$.

It is natural to require that the symplectic form is **non-degenerate**. We are then led to the converse statement: If given X we have $(\omega_{\partial M})_\phi(X, Y) = 0$ for all induced Y, then X itself must be induced. This means, L_M induces a **coisotropic** submanifold of $L_{\partial M}$.

Taking both statements together yields,

L_M induces a **Lagrangian** submanifold of $L_{\partial M}$.

Geometric quantization: Prequantization

Geometric quantization is designed to output the structures of the **standard formulation** of quantum theory, i.e., a Hilbert space of states and an operator algebra of observables acting on it. Its main input is the space L of solutions of the classical theory in spacetime with its symplectic structure ω. It proceeds roughly in two steps:

1. We consider a hermitian line bundle B over L with a connection ∇ that has curvature 2-form ω. Define the **prequantum** Hilbert space H as the space of square-integrable sections with inner product

$$\langle s', s \rangle = \int (s'(\eta), s(\eta))_\eta \, \mathrm{d}\mu(\eta).$$

Here the measure $\mathrm{d}\mu$ is given by the $2n$-form $\omega \wedge \cdots \wedge \omega$ if L has dimension $2n$. Classical observables, i.e., functions on L, act naturally as operators on H with the "correct" commutation relations.

2 This Hilbert space is too large. Choose in each complexified tangent space $(T_\phi L)^{\mathbb{C}}$ a Lagrangian subspace P_ϕ with respect to ω_ϕ. We then restrict H to those sections s of B such that

$$\nabla_{\overline{X}} s = 0,$$

if $X_\phi \in P_\phi$ for all $\phi \in L$. This is called a **polarization**. The subspace $\mathcal{H}$ of H obtained in this way is the Hilbert space of states. Not all observables are well defined on it as they might not leave the subspace $\mathcal{H} \subseteq H$ invariant.

Kähler polarization

We are interested in a **Kähler polarization**. Then P_ϕ is determined by a complex structure J_ϕ in $T_\phi L$ that is compatible with ω_ϕ. J_ϕ satisfies $J_\phi \circ J_\phi = -1$ and $\omega_\phi(J_\phi X, J_\phi Y) = \omega_\phi(X, Y)$. Then

$$P_\phi = \{X \in (T_\phi L)^{\mathbb{C}} : \mathrm{i}X = J_\phi X\}.$$

J_ϕ yields a real inner product on $T_\phi L$:

$$g_\phi(X_\phi, Y_\phi) := 2\omega_\phi(X_\phi, J_\phi Y_\phi).$$

We shall require g_ϕ to be positive definite. We also obtain a complex inner product on $T_\phi L$ viewed as a complex vector space:

$$\{X_\phi, Y_\phi\}_\phi := g_\phi(X_\phi, Y_\phi) + 2\mathrm{i}\omega_\phi(X_\phi, Y_\phi).$$

The Hilbert space $\mathcal{H}$ obtained from H through a Kähler polarization is also called the **holomorphic representation**.

Linear field theory

To be able to deal with the field theory case where L is generically infinite-dimensional we restrict ourselves to the simplest setting of linear field theory. That is, we take L to be a real vector space and the symplectic form ω to be invariant under translations in L. Not much is known beyond this setting.

Then, L can be naturally identified with its tangent space. Moreover, the symplectic form ω, the complex structure J, the real and complex inner products g, $\{\cdot,\cdot\}$ all become structures on the vector space L. The line bundle B becomes trivial and its section (the elements of H) can be identified with complex functions on L. For a Kähler polarization the elements of the subspace $\mathcal{H} \subseteq H$ are precisely the **holomorphic** functions on L. Moreover, the inner product formula simplifies,

$$\langle \psi', \psi \rangle = \int \overline{\psi'(\eta)}\psi(\eta) \exp\left(-\frac{1}{2}g(\eta,\eta)\right) \mathrm{d}\mu(\eta).$$

The measure

What is the measure $\mathrm{d}\mu$?

It turns out that on an infinite-dimensional vector space L no translation-invariant measure exists. Instead, we should look for a **Gaussian measure**

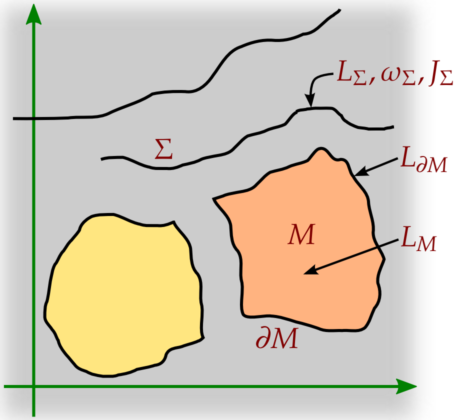
$$\mathrm{d}\nu \approx \exp\left(-\frac{1}{2}g(\eta,\eta)\right)\mathrm{d}\mu.$$

However, not even that exists on the Hilbert space L. The measure does exist if we extend L to a larger vector space $\hat{L}$. Concretely ν and $\hat{L}$ can be constructed as an inductive limit of finite-dimensional quotient spaces of L. It turns out that $\hat{L}$ can also be identified with the algebraic dual of the topological dual of L.

A priori, wave functions are thus really functions of $\hat{L}$ rather than on L. But, a function that is square-integrable on $\hat{L}$ and holomorphic is completely determined by its values on L. This allows us to "forget" about $\hat{L}$ to some extent.

Bosonic semiclassical linear field theory

Spacetime is modeled by a collection of **hypersurfaces** and **regions**.



To these geometric structures associate the classical data,

- per **hypersurface** Σ :
  a symplectic vector space $(L_\Sigma, \omega_\Sigma)$ of **solutions** near Σ,
- per **region** M :
  a **Lagrangian subspace** $L_M \subseteq L_{\partial M}$ of **solutions** in M.

In addition,

- per **hypersurface** Σ :
  a **complex structure** J_Σ .

It follows that $L_{\partial M} = L_M \oplus_{\mathbb{R}} J_{\partial M} L_M$ is an orthogonal sum.

Holomorphic quantization: State spaces

For each hypersurface Σ we define a Hilbert space of states $\mathcal{H}_\Sigma$ by using the geometric quantization prescription. Thus, $\mathcal{H}_\Sigma$ is a space of holomorphic functions on L_Σ with the inner product,

$$\langle\psi',\psi\rangle_\Sigma := \int_{\hat{L}_\Sigma} \overline{\psi'(\phi)}\psi(\phi)\,\mathrm{d}\nu_\Sigma(\phi).$$

Holomorphic quantization: Amplitudes

For each region M we define the linear amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$ by

$$\rho_M(\psi) := \int_{\hat{L}_M} \psi(\phi)\, \mathrm{d}\nu_M(\phi).$$

Here $\hat{L}_M$ is an extension of L_M and ν_M is a Gaussian measure on $\hat{L}_M$, depending on $g_{\partial M}$ that heuristically takes the form

$$\mathrm{d}\nu_M \approx \exp\left(-\frac{1}{4} g_M(\eta, \eta)\right) \mathrm{d}\mu$$

with μ a (fictitious) translation-invariant measure.

It can be shown that this prescription is here equivalent to the Feynman path integral prescription.

Holomorphic quantization: main result

We obtain a quantum theory in terms of the data of the GBF. [RO 2010]

Theorem

With an additional integrability assumption, the GBF core axioms are satisfied by this quantization prescription.

The quantization scheme may be viewed (in various ways) as a **functor** from classical field theories to general boundary quantum field theories.

This scheme can be generalized to **affine field theories**. [RO 2011]

Coherent States

The Hilbert spaces $\mathcal{H}_\Sigma$ are reproducing kernel Hilbert spaces and contain coherent states of the form

$$K_\xi(\phi) = \exp\left(\frac{1}{2}\{\xi, \phi\}_\Sigma\right)$$

associated to classical solutions $\xi \in L_\Sigma$. They have the reproducing property,

$$\langle K_\xi, \psi \rangle_\Sigma = \psi(\xi),$$

and satisfy the completeness relation

$$\langle \psi', \psi \rangle_\Sigma = \int_{\hat{L}_\Sigma} \langle \psi', K_\xi \rangle_\Sigma \langle K_\xi, \psi \rangle_\Sigma \, \mathrm{d}\nu_\Sigma(\xi).$$

They can be thought of as representing quantum states that approximate specific classical solutions.

Universal amplitude formula

Remarkably, the amplitude can be written down in closed form. [RO 2010]

- Consider a region M.
- $\xi \in L_{\partial M}$ a solution on the boundary of M.
- Decompose uniquely $\xi = \xi^{\mathrm{c}} + \xi^{\mathrm{n}}$ into the classically allowed ($\xi^{\mathrm{c}} \in L_M$) and forbidden ($\xi^{\mathrm{n}} \in J_{\partial M} L_M$) parts.
- The amplitude for the associated normalized coherent state $\tilde{K}_\xi$ is:

$$\boxed{\rho_M(\tilde{K}_\xi) = \exp\left(\mathrm{i}\,\omega_{\partial M}(\xi^{\mathrm{n}}, \xi^{\mathrm{c}}) - \frac{1}{2} g_{\partial M}(\xi^{\mathrm{n}}, \xi^{\mathrm{n}})\right)}$$

This has a simple and compelling physical interpretation.

Fermionic field theory (I)

Starting with a **Lagrangian density** Λ we obtain a **symplectic form** $\tilde{\omega}_\Sigma$ associated to any hypersurface Σ as in the bosonic case.

A fermionic field is generally a section of a **complex vector bundle** (associated with the spin bundle). The associated complex structure can be used to produce a **symmetric bilinear form** g_Σ from $\tilde{\omega}_\Sigma$. This (and not $\tilde{\omega}_\Sigma$) is the "correct" object to encode fermionic field theory:

$$g_\Sigma(X, Y) = 2\tilde{\omega}_\Sigma(X, \mathrm{i}Y)$$

(g_Σ can be also be derived directly by already taking into account the "anti-commuting" nature of the fermionic field at the classical level.)

The symmetric form g_Σ arises from the integral of a $(d-1)$-form on a hypersurface. Its sign thus depends on **orientation**: $\boxed{g_{\overline{\Sigma}} = -g_\Sigma}$.

Fermionic field theory (II)

As in the bosonic case the additional ingredient for the **geometric quantization** on a hypersurface is the **complex structure** $J_\Sigma : L_\Sigma \to L_\Sigma$. This has to satisfy $J_\Sigma^2 = -\mathbf{1}$ and $g_\Sigma(J_\Sigma \cdot, J_\Sigma \cdot) = g_\Sigma(\cdot, \cdot)$.

As in the bosonic case, the complex structure encodes a kind of global orientation. Its sign thus depends on **orientation**: $\boxed{J_{\overline{\Sigma}} = -J_\Sigma}$.

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Let M be a region and L_M the space of solutions in M. Then we have a natural map $L_M \to L_{\partial M}$ by "forgetting" the solution in the interior of M. The following key property encodes the **classical dynamics**.

L_M induces a **hypermaximal neutral subspace** of $L_{\partial M}$:

- $g_{\partial M}(\phi, \phi') = 0$ for all $\phi, \phi' \in L_M$.
- If $\phi \notin L_M$ then there is $\phi' \in L_M$ such that $g_{\partial M}(\phi, \phi') \neq 0$.

There is a **compatibility condition** between $J_{\partial M}$ and L_M.

The appearance of Krein spaces

Similar to the bosonic case, the structures associated to a hypersurface induce a **complex inner product**:

$$\omega_\Sigma(\phi, \phi') := \frac{1}{2} g_\Sigma(J_\Sigma \phi, \phi')$$

$$\{\phi, \phi'\}_\Sigma := g_\Sigma(\phi, \phi') + 2\mathrm{i}\omega_\Sigma(\phi, \phi')$$

But recall that g_Σ changes sign under orientation change and $g_{\partial M}$ is partially null.

Thus, the spaces L_Σ are not in general Hilbert spaces. Instead, they are **Krein spaces**, a special version of **indefinite inner product spaces** that decompose as

$$L_\Sigma = L_\Sigma^+ \oplus L_\Sigma^-.$$

Here, L_Σ^+ is **positive definite** and L_Σ^- is **negative definite**. (This decomposition also provides for a topology on L_Σ.)

Fermionic semiclassical linear field theory

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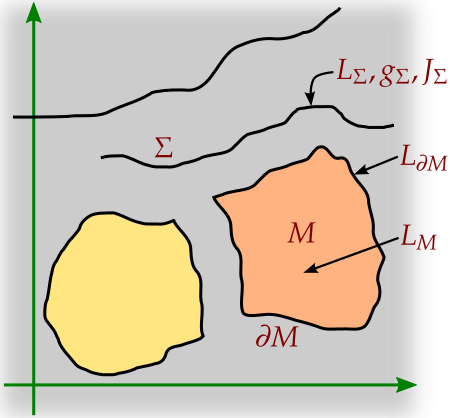

To these geometric structures associate the classical data,

- per **hypersurface** Σ :
  a **real Krein space** (L_Σ, g_Σ),
- per **region** M :
  a **hypermaximal neutral subspace** $L_M \subseteq L_{\partial M}$.

In addition,

- per **hypersurface** Σ :
  a **complex structure** J_Σ .

Structure of quantum field theory in the GBF

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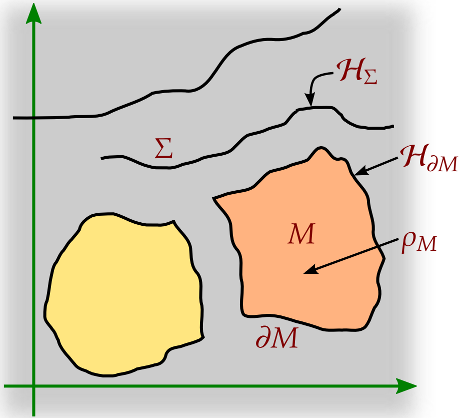

To these geometric structures associate the quantum data,

- per **hypersurface** Σ : an **f-graded Krein space** $\mathcal{H}_\Sigma$,
- per **region** M : a linear **f-graded amplitude map** $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$.

Compared to the purely bosonic case we have a $\mathbb{Z}_2$-grading called **f-grading** on all structures. Moreover, instead of **Hilbert spaces** we have **Krein spaces**.

Fock space (I)

We distinguish bosonic and fermionic case via

$$\kappa := 1 \quad \text{in the bosonic case,} \qquad \kappa := -1 \quad \text{in the fermionic case.}$$

Given a Krein space L, the **Fock space** $\mathcal{F}(L)$ over L is the completion of an $\mathbb{N}_0$-graded Krein space,

$$\mathcal{F}(L) = \widehat{\bigoplus_{n=0}^{\infty}} \mathcal{F}_n(L),$$

$$\mathcal{F}_n(L) := \{\psi : L \times \cdots \times L \to \mathbb{C} \ n\text{-lin. cont. } : \psi \circ \sigma = \kappa^{|\sigma|}\psi, \forall \sigma \in S^n\}.$$

There is a natural $\mathbb{Z}_2$-grading. In the bosonic case it is trivial, i.e., $|\psi| = 0$ for all $\psi \in \mathcal{F}(L)$. In the fermionic case it is,

$$|\psi| := \begin{cases} 0 & \text{if } \psi \in \mathcal{F}_n(L), n \text{ even} \\ 1 & \text{if } \psi \in \mathcal{F}_n(L), n \text{ odd.} \end{cases}$$

Fock space (II)

Given $\xi_1, \ldots, \xi_n \in L$ define a **generating state** in $\mathcal{F}_n(L)$ as

$$\psi[\xi_1, \ldots, \xi_n](\eta_1, \ldots, \eta_n) := \frac{1}{n!} \sum_{\sigma \in S^n} \kappa^{|\sigma|} \prod_{i=1}^{n} \{\xi_i, \eta_{\sigma(i)}\}.$$

The inner product in Fock space is determined by the inner product of generating states,

$$\langle \psi[\eta_1, \ldots, \eta_n], \psi[\xi_1, \ldots, \xi_n] \rangle := 2^n \sum_{\sigma \in S^n} \kappa^{|\sigma|} \prod_{i=1}^{n} \{\xi_i, \eta_{\sigma(i)}\}.$$

This makes $\mathcal{F}(L)$ into a **Krein space** as well.

Fock quantization: State spaces

For each hypersurface Σ we define the corresponding **state space** $\mathcal{H}_\Sigma$ to be the Fock space $\mathcal{F}(L_\Sigma)$.

For all $n \in \mathbb{N}_0$ define $\iota_{\Sigma,n} : \mathcal{F}_n(L_\Sigma) \to \mathcal{F}_n(L_{\overline{\Sigma}})$ by,

$$(\iota_{\Sigma,n}(\psi))(\xi_1, \dots, \xi_n) := \overline{\psi(\xi_n, \dots, \xi_1)}.$$

Taking these maps together for all $n \in \mathbb{N}_0$ defines $\iota_\Sigma : \mathcal{F}(L_\Sigma) \to \mathcal{F}(L_{\overline{\Sigma}})$.

A decomposition $\Sigma = \Sigma_1 \cup \Sigma_2$ induces a direct sum of Krein spaces $L_\Sigma = L_{\Sigma_1} \oplus L_{\Sigma_2}$. This induces an isomorphism of Fock spaces

$$\tau_{\Sigma_1,\Sigma_2;\Sigma} : \mathcal{F}(L_{\Sigma_1}) \otimes \mathcal{F}(L_{\Sigma_2}) \to \mathcal{F}(L_\Sigma).$$

This also yields the f-graded transposition,

$$\mathcal{F}(L_{\Sigma_1}) \otimes \mathcal{F}(L_{\Sigma_2}) \to \mathcal{F}(L_{\Sigma_2}) \otimes \mathcal{F}(L_{\Sigma_1}) \quad : \quad \psi_1 \otimes \psi_2 \mapsto (-1)^{|\psi_1|+|\psi_2|} \psi_2 \otimes \psi_1.$$

Fock quantization: Amplitudes

Given a region M we recall the real orthogonal decomposition $L_{\partial M} = L_M \oplus J_{\partial M} L_M$ giving rise to the map $u_M : L_{\partial M} \to L_{\partial M}$,

$$u_M(\xi + J_{\partial M}\eta) = \xi - J_{\partial M}\eta, \quad \forall \xi, \eta \in L_M.$$

The **amplitude** for a generating state is now defined as,

$$\rho_M(\psi[\xi_1, \ldots, \xi_{2n}]) := \frac{1}{n!} \sum_{\sigma \in S^{2n}} \kappa^{|\sigma|} \prod_{j=1}^{n} \{\xi_{\sigma(j)}, u_M(\xi_{\sigma(2n+1-j)})\}_{\partial M}.$$

The amplitude vanishes for states with odd particle number.

Fock quantization: Main result

This **quantization scheme** yields the data of a quantum theory in terms of the GBF. [RO 2012]

Theorem

With an additional integrability assumption, the GBF core axioms as well as the vacuum axioms are satisfied.

The quantization prescription may be viewed (in various ways) as a **functor** from semiclassical field theories to general boundary quantum field theories.

In the bosonic case the Fock quantization scheme is **equivalent** to the Holomorphic quantization scheme.

Probabilities and superselection (I)

We recall the probability rule for the bosonic GBF, where all state spaces are **Hilbert spaces**. A measurement is determined by two subspaces of $\mathcal{H}_{\partial M}$,

- $\mathcal{S}$, representing the **preparation** and
- $\mathcal{A} \subseteq \mathcal{S}$, representing the **question** asked .

The probability for an affirmative answer is then,

$$P(\mathcal{A}|\mathcal{S}) = \frac{\sum_{i\in J} |\rho_M(\xi_i)|^2}{\sum_{i\in I} |\rho_M(\xi_i)|^2}.$$

Here $\{\xi\}_{i\in I}$ is an ON-basis of $\mathcal{S}$ and $\{\xi\}_{i\in J}$ is an ON-basis of $\mathcal{A}$, with $J \subseteq I$.

Probabilities and superselection (II)

The very same formula works for **Krein spaces**. But there are some differences.

The notion of ON-basis is more restrictive in the Krein space case. It implies that the subspaces $\mathcal{S}$ and $\mathcal{A}$ must decompose as direct sums $\mathcal{S} = \mathcal{S}^+ \oplus \mathcal{S}^-$ and $\mathcal{A} = \mathcal{A}^+ \oplus \mathcal{A}^-$, where $\mathcal{S}^\pm, \mathcal{A}^\pm \subseteq \mathcal{H}^\pm_{\partial M}$. This amounts to a **signature superselection rule**.

In the fermionic case this superselection rule is not invariant under **orientation change**. But the physics should be. But for fermionic theories there is also the **fermionic superselection rule** [Wick, Wightman, Wigner 1952] that forbids the mixing of states with even and odd fermion number. This amounts to requiring decompositions $\mathcal{S} = \mathcal{S}_0 \oplus \mathcal{S}_1$ and $\mathcal{A} = \mathcal{A}_0 \oplus \mathcal{A}_1$ in terms of the **f-grading** of $\mathcal{H}_{\partial M}$.

In combination with the fermionic superselection rule, the signature superselection rule becomes **orientation invariant**.

Further remarks on fermions

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  On a globally hyperbolic spacetime for **Dirac fermions** the inner product on **spacelike hypersurfaces** is always **definite**. With a (usually implicit) global choice of orientation it is always **positive definite**. Consequently the associated state spaces are all **Hilbert space**.

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- Time emerging

  It turns out that the map $u_M : L_{\partial M} \to L_{\partial M}$ also encodes a notion of **evolution in time**. The quantum theory inherits this notion. This is a purely algebraic phenomenon, independent of a spacetime metric. This may suggest on **emergent notion of time** in a theory of quantum gravity with fermions.

Most recent developments

Everything I have shown you so far is based on a formalism involving **Hilbert** or **Krein spaces**, **amplitude maps** and **observable maps**. This was inspired by the **path integral** and the standard formulation in terms of pure states. I refer to this now as the **amplitude formalism**.

There is a new formalism that involves "mixed states". This **positive formalism** is obtained by "taking the modulus square" of the **amplitude formalism**. [RO 2012]

Properties of the Positive Formalism

- Remarkably, it still admits the same **composition properties** as the **path integral**!
- Focuses on **operationally relevant information** and eliminates **unphysical data** (phases etc.).
- Probability interpretation simpler and conceptually clearer.
- All models expressed in the amplitude formalism can be translated into the positive formalism (functorially).
- Admits general **quantum operations** and opens the GBF to **quantum information theory**.
- More freedom to formulate quantum theories and quantization schemes.
- Appears necessary to overcome the **state locality problem** in QFT. [RO 2013]