LECTURE 1: What is wrong with the standard formulation of quantum theory?

Robert Oeckl

IQG-FAU & CCM-UNAM

IQG FAU Erlangen-Nürnberg
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Outline

1. Classical physics
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   - Reality in quantum physics
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3. Conceptual limitations
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Overview

Historically, quantum theory was first developed in a non-relativistic context, modeled on an **analogy with non-relativistic classical mechanics**. This imprinted a special role of time on its very foundations as well as a lack of manifest locality.

This precludes the application of quantum theory in a general relativistic context.

Overview

Historically, quantum theory was first developed in a non-relativistic context, modeled on an **analogy with non-relativistic classical mechanics**. This imprinted a special role of time on its very foundations as well as a lack of manifest locality.

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To clarify these issues it is crucial to understand the radically different ways that reality is modeled in classical versus quantum physics.

Reality in classical physics

Distinguishing features of classical physics are:

local realism

A physical theory provides a direct description of objective reality as localized in space and time.

determinism

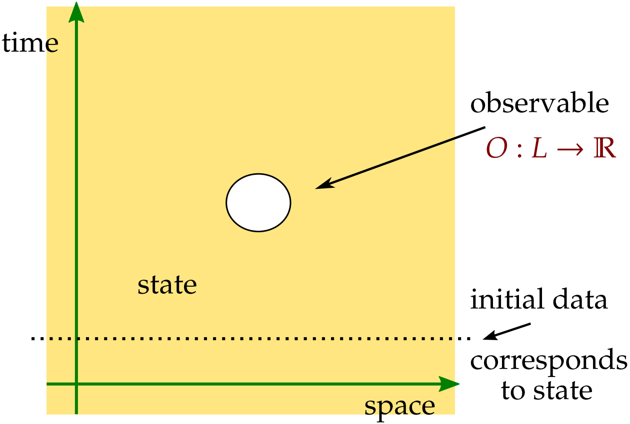
Given complete knowledge of physical reality at some instant of time allows the extrapolation in principle of this knowledge to all of the past and future. In a relativistic context determinism is sharpened to causality.

independence

The observed reality is independent of the act of observation.

Measurement in classical physics

The system is determined by a dynamical law and a state.



State space L.

Measurements yield objective information about the state

States are global in spacetime

Non-relativistic mechanics

In **non-relativistic** classical mechanics it is convenient to use:

- A **Phase Space** P of **initial data**, providing a complete description of physics at an instant of time. It carries a symplectic structure ω.
- A **Hamiltonian** $H : P \to \mathbb{R}$ yielding a complete description of the time-evolution in phase space by determining a **flow** X_H on P via

$$\mathrm{d}H(Y) = 2\omega(Y, X_H).$$

- **Observables** $O : P \to \mathbb{R}$ characterizing measurable quantities of the system. Their time-evolution is induced by the flow X_H.

At a given moment in time a state may be identified with initial data, yielding an identification of the spaces P and L.

Quantum theory: standard formulation

The **standard formulation** of quantum theory is modeled after non-relativistic classical mechanics:

- A **State Space** $\mathcal{H}$ (Hilbert space) in analogy to the phase space, giving information about physics at an instant of time.
- A **Hamiltonian** $H : \mathcal{H} \to \mathcal{H}$ (hermitian operator) in analogy to the classical Hamiltonian, describing the evolution in time of states.
- **Observables** $O : \mathcal{H} \to \mathcal{H}$ (hermitian operators) in analogy to the classical observables, describing measurement processes.

Reality in quantum physics

- A state encodes information about the system at a time.
- This information is maximal in the sense that there is no additional information that could improve predictions of future measurement outcomes.
- Even the complete knowledge of a state only allows probabilistic predictions of future measurement outcomes.
- In general, a measurement modifies a state.
- The observer must be *external* to the system and is subject to a *classical* description.
- Assuming that a state is an image of the reality of the system leads to the conclusion that this reality is non-local. (collapse of the wavefunction)

Time-evolution and probability

Standard formulation

In the absence of a measurement a state $\psi \in \mathcal{H}$ evolves from time t_1 to time t_2 via

$$\psi \mapsto U(t_1, t_2)\psi \quad \text{where} \quad U(t_1, t_2) := e^{-\mathrm{i}H(t_2 - t_1)}$$

is the unitary **time-evolution operator**.

Measurements are described by observables. Consider a yes/no question. This is represented by an orthogonal projection operator $P : \mathcal{H} \to \mathcal{H}$. Given a normalized initial state ψ the probability for the outcome

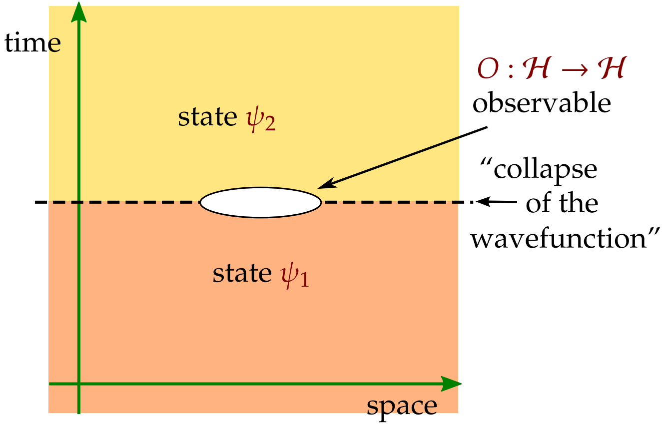
- **yes** is: $\|P\psi\|^2$ with resulting state $P\psi/\|P\psi\|$
- **no** is: $\|(1-P)\psi\|^2$ with resulting state $(1-P)\psi/\|(1-P)\psi\|$

The measurement process is probabilistic, not deterministic.
Measurements change states instantaneously.

Measurement in quantum physics I

Standard formulation

The system is determined by a dynamical law and exhibits a sequence of states. The state space is a Hilbert space $\mathcal{H}$.



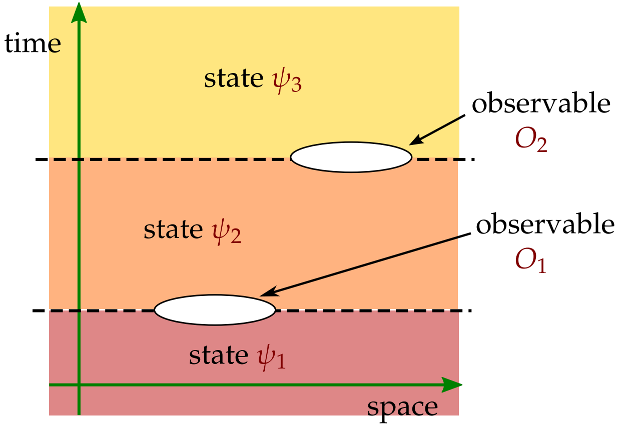
Measurements modify the state and are probabilistic

States are global in space but local in time

Time plays a special role!

Measurement in quantum physics II

Standard formulation



The operator product $O_2 \cdot O_1$ encodes joint measurement. Its order is the temporal order of measurements.

E.g. $[Q, P] = i\hbar$

Time plays a special role!

The special role of time

Conclusion:

In contrast to classical physics, the standard formulation of quantum theory requires a predetermined notion of time to make sense.

A non-relativistic setting provides such a notion of time.

Compatibility with special relativity

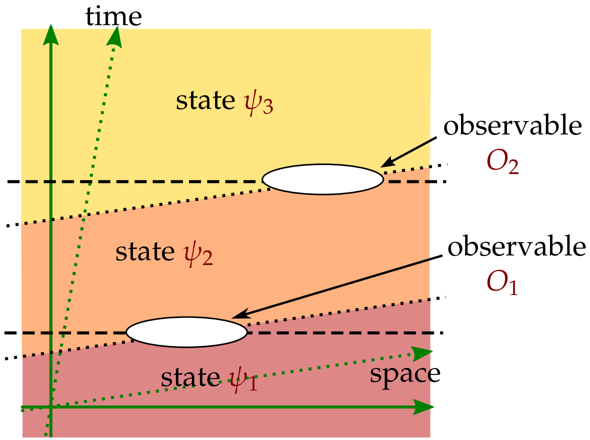
However, we know that physics is **relativistic**.

In **special relativity** there is no single predetermined notion of time, but one for each **inertial frame**. We can achieve compatibility by ensuring that different choices of inertial frames lead to equivalent results. To this end observables are **labeled** by spacetime points. If two spacetime points x, y are **spacelike** separated their temporal ordering in different frames can be different. To avoid inconsistencies we must then require that observables $O_1(x)$ and $O_2(y)$ **commute**, i.e.,

$$O_1(x)O_2(y) = O_2(y)O_1(x).$$

Measurement in boosted frame

Standard formulation

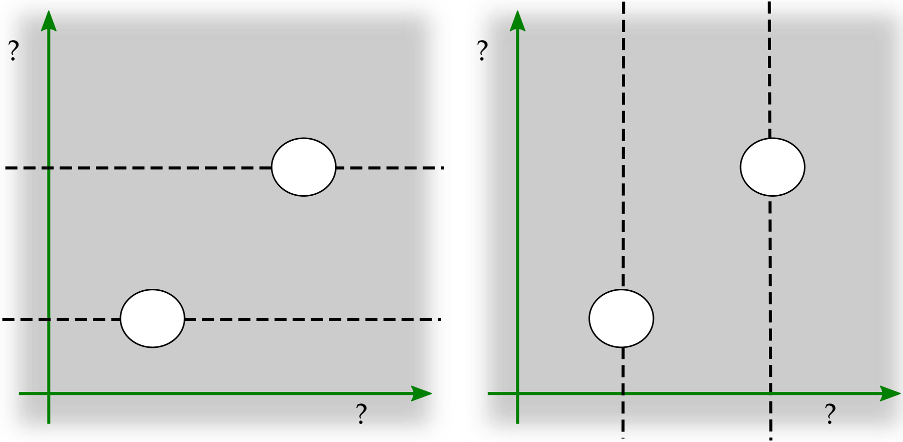


Even if the temporal order of measurements remains the same, the assignment of states to spacetime regions changes.

Time plays a special role!

Quantum measurement without spacetime metric?

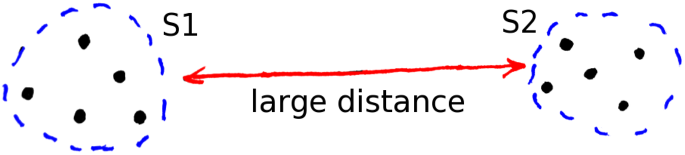
If spacetime is dynamical, as in a **general relativistic** setting, there is no a priori metric "separating" space and time. What do we do then?



Here, the standard formulation of quantum theory breaks down.

Locality in the standard formulation

In a fundamental quantum theory a state is a priori a state of the universe. But, we cannot hope to be able to describe the universe in all its details. We need to be able to describe physics locally. In quantum field theory this is achieved dynamically, using the background metric. Causality and cluster decomposition mean that the S-matrix factorizes, $S = S_1 S_2$:



We can thus successfully describe a local system as if it was alone in an otherwise empty Minkowski universe.

In a general relativistic setting we have no a priori metric and this dynamical implementation of locality fails.

How do we do quantum gravity? (I)

Traditionally three lines of attack have been followed:

1. Do what you can do: An approximate fixed metric spacetime geometry is postulated at asymptotic infinity. Observations take place exclusively in this region. There are no problems in applying the standard formulation there. (Perturbative Quantum Gravity, String Theory)

Predictive power is very probably lost. It is questionable whether the quality of the approximation can be controlled, even in principle.

How do we do quantum gravity? (II)

Traditionally three lines of attack have been followed:

2. Just go for it: In order to describe the "deep" quantum gravity regime as well a new physical interpretation of the mathematical objects of the standard formulation must be provided. On top of this the locality problem needs to be solved. (Quantum Geometrodynamics, Loop Quantum Gravity)

So far no satisfactory interpretation has been proposed. There might be none.

How do we do quantum gravity? (III)

Traditionally three lines of attack have been followed:

3. Quantum theory is wrong: Quantum theory as we know it is fundamentally limited and must be replaced by some different underlying theory. Known physics is modified. (Causal sets, Gravity induced collapse models)

There is no evidence for violations of quantum theory as we know it. Also, it is difficult to reinvent physics from scratch.

How do we do quantum gravity? (IV)

OR

The standard formulation is limited, not quantum theory itself: There is a more suitable and more fundamental formulation of quantum theory, free of these limitations. This is what we should understand and use instead.

The general boundary formulation

The **general boundary formulation** is a novel formulation of quantum theory

- where time plays no special role
- that is metric **background independent**
- that is manifestly **spacetime local**
- that embraces known quantum physics

It is still under construction, so you can contribute!

This is what this lecture series is about.