

# NON-MINIMAL YANG-MILLS-HIGGS FIELDS OVER THE FOUR DIMENSIONAL TORUS

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**ABSTRACT.** The Yang–Mills–Higgs functional for  $SU(2)$ -connections on the trivial  $\mathbf{C}^2$  bundle over the four dimensional torus is studied. We find an infinite number of non-minimal solutions to the coupled Yang–Mills–Higgs equations. These solutions are derived in an explicit way using symmetry and applying algebraic procedures.

## 1. Introduction.

Given a vector bundle  $E$  with structure group  $G$  over a Riemannian manifold  $M$ , for every pair  $(A, \phi)$ , where  $A$  is a  $G$ -connection on  $E$  and  $\phi$  is a section of  $E$ , the Yang–Mills–Higgs functional is:

$$(1.1) \quad YMH(A, \phi) = \frac{1}{2} \int_M \left[ \frac{1}{2} |F|^2 + |D\phi|^2 + \frac{\lambda}{2} |\phi|^4 - m^2 |\phi|^2 \right] d\mu.$$

Here  $F$  and  $D$  are the curvature and the covariant derivative associated with  $A$ , respectively,  $m$  is the mass parameter, and  $\lambda$  is a real positive constant.

Historically, the  $YMH$  functional and its associated variational equations, appeared in the theory of superconductivity of V. Ginsburg and L. Landau in 1950 [4], and in the field of high energy physics as the bosonic component of the standard model for the strong and electroweak interactions [8]. A complete description of the  $YMH$  fields on  $\mathbf{R}^2$  (vortices) has been given by A. Jaffe and C. Taubes in [6]. For the case of monopoles in three dimensional manifolds (which correspond to making the potential term in (1.1) identically zero), their classification and moduli spaces are also known in several cases [5]. In  $\mathbf{R}^d$  of dimension  $d$  at least five it is known that there are no non-trivial finite action solutions, and that in the

four dimensional case a finite action solution is gauge equivalent to a pure Yang-Mills field, [6] p. 32. More recently S. Bradlow has studied Hermitian *YMH* fields for complex vector bundles over closed Kähler manifolds [3], and has obtained important relations between these fields and the notion of  $\phi$ -stability for algebraic vector bundles. This author also answered questions related to the existence of solutions and the moduli space of the solutions on line bundles. However, and in contrast with the pure Yang-Mills case where one has Taubes existence theorems for instantons, and the ADHM description of the moduli spaces of instantons [5], for the case of *YMH* fields over compact four-dimensional Riemannian manifolds very little is known to date. An important development along this line is contained in the work of T. H. Parker [10], who recently has given a topological proof of existence of *YMH* fields on compact Riemannian four-manifolds based on the methods of equivariant Morse Theory. This work asserts the existence of *YMH* fields under the action of some compact Lie group (under appropriate conditions in the action). Nevertheless there is so far no knowledge about the nature and structure of these solutions, and no coupled explicit solutions are known for this problem [10]. Thus the issue of classification of the *YMH* fields is far from being solved. On the other hand, the question of the existence of non-minimal Yang-Mills fields was answered affirmatively in [14] by L. Sibner, R. Sibner and K. Uhlenbeck who have found examples of non-minimal Yang-Mills fields on the standard  $S^4$ . Since then many others examples for different manifolds haven been found, see e. g. [15], [13], [10], [2].

The purpose of this work is to exhibit what we believe are the first non-minimal and non-trivial explicit solutions to the coupled *YMH*-equations. The solutions are derived by making use of the theory of invariant connections and equivariant sections. Our main result is contained in the following theorem the context of which is proven in section 3:

**Theorem 1.** *Let  $(T^4, g)$  be a four dimensional flat torus which is the Riemannian product of circles. In the trivial  $\mathbb{C}^2$  vector bundle over  $(T^4, g)$  there exist  $SU(2)$ -solutions to the coupled *YMH* equations which are not absolute minima of the *YMH* functional. These solutions can be derived in an explicit way by algebraic procedures.*

Basically, since  $(T^4, g)$  are Kählerian manifolds, we use  $SU(2)$ -connections and put the usual Hermitian structure on the vector bundle. Thus, we are

working essentially in the same context of the problem consider in [3]. An important difference is that we work with the real *YMH* equations without the use of the Kählerian structure and not just with the equations of the minima as is done in [3] (see Corollary 2.3 in that paper). Also in [3] the existence of the solutions of the vortex equations depends on some conditions relating the mass parameter, the volume of the base manifold and the slope of the line bundle. Our results also lead to a restriction on the values of the mass parameter, although they are of a completely different nature as they provide a lower bound for the mass in terms of  $\lambda$  and the Higgs parameters (see Remark 2 and Equation (4.10)).

In section 2 we construct invariant connections and equivariant sections. Section 3 contains the proof of the Theorem, and in Section 4 some comments are given relating to the gauge classification of the solutions. Section 5 shows that some of the solutions obtained are not stable.

## 2. Invariant pairs $(\Lambda, \phi)$ .

Let  $T^4$  be the four dimensional torus, with the usual family of flat Riemannian metrics:

$$(2.1) \quad g = \sum r_i^2 d\theta_i \otimes d\theta_i, \quad 1 \leq i \leq 4, \quad r_i \in \mathbf{R}, \quad \theta_i \in S^1 = \mathbf{R}/4\pi\mathbf{Z},$$

where we have made the unusual choice of the  $4\pi$  factor in the definition of  $S^1$ , so as to simplify our later calculations of  $\phi$ . We consider  $T^4$  as a homogeneous space under the action of itself. Let  $P$  be the trivial  $SU(2)$ -bundle over  $T^4$ . According to H. C. Wang, [7] p. 103, all linear maps

$$(2.2) \quad \Lambda : \mathbf{R}^4 \rightarrow SU(2), \quad \Lambda = \begin{pmatrix} a_1 \dots a_4 \\ b_1 \dots b_4 \\ c_1 \dots c_4 \end{pmatrix},$$

define an  $SU(2)$ -connection in  $P$  which is invariant under the trivial  $T^4$ -action on  $P$ . We make the identification  $\mathbf{R}^2 = \mathbf{C}$ , hence  $GL(2, \mathbf{C}) \subset GL(4, \mathbf{R})$ , and also impose on  $\mathbf{R}^{2n} = \mathbf{C}^n$  the additional structure of the usual Hermitian product. Consider the unitary natural representation  $\rho$  of  $SU(2) \subset GL(2, \mathbf{C})$ , where  $\rho_*$  is given by  $\{\frac{-\sqrt{-1}}{2}\sigma_j\}$ , and  $\{\sigma_j\}$  are the Pauli matrices, and let  $E = P \times_{SU(2)} \mathbf{C}^2$ , be the associated trivial vector bundle. The  $T^4$ -action is lifted to  $E$  by means of the representation

$$(2.3) \quad \mu : T^4 \rightarrow SU(2) \subset GL(2, \mathbf{C})$$

$$(\theta_1, \dots, \theta_4) \mapsto \begin{pmatrix} \exp [\sqrt{-1} \sum (\theta_i/2)] & 0 \\ 0 & \exp [-\sqrt{-1} \sum (\theta_i/2)] \end{pmatrix}.$$

Note that  $\mu(4\pi e_i) = Id$ , where  $\{e_i\}$  is the usual basis of  $\mathbf{R}^4$ , hence the action of  $\mu$  on the trivial bundle over  $\mathbf{R}^4$  descends to the trivial bundle over  $T^4$ . This action of  $T^4$  is by unitary transformations, since we have added the usual Hermitian structure to  $E$ . We are interested in studying  $T^4$ -equivariant Higgs fields on  $E$ , i. e. maps  $\phi: T^4 \rightarrow \mathbf{C}^2$  such that

$$(2.4) \quad \phi(\bar{\theta} \cdot \theta) = \mu(\bar{\theta}) \phi(\theta) \quad \bar{\theta}, \theta \in T^4.$$

By solving for the real components  $\phi = (\phi_1, \dots, \phi_4)$  we get :

$$(2.5) \quad \phi_1(\theta) = x_1 \cos(\sum (\theta_i/2)) - x_2 \sin(\sum (\theta_i/2)) ,$$

$$(2.6) \quad \phi_2(\theta) = -x_1 \sin(\sum (\theta_i/2)) - x_2 \cos(\sum (\theta_i/2)) ,$$

$$(2.7) \quad \phi_3(\theta) = x_3 \cos(\sum (\theta_i/2)) - x_4 \sin(\sum (\theta_i/2)) ,$$

$$(2.8) \quad \phi_4(\theta) = x_3 \sin(\sum (\theta_i/2)) + x_4 \cos(\sum (\theta_i/2)) ,$$

where  $(x_1, \dots, x_4) \in \mathbf{R}^4$  are considered as parameters, and  $\theta = (\theta_1, \dots, \theta_4) \in T^4$ . We say that a pair  $(\Lambda, \phi)$  is  $T^4$ -invariant if it is constructed as above.

Note that since we do not use the trivial action to define  $\phi$ , our equivariant Higgs fields are not constant sections and do not result in trivial solutions (in the sense of [10]) to the *YMH* equations. The set of all these flat metrics and  $T^4$ -invariant pairs  $\{g, (\Lambda, \phi)\}$  form a manifold diffeomorphic to  $\mathbf{R}^{20}$ . We shall next look for critical values of the *YMH* functional in this space.

### 3. Variational equations and solutions.

The critical points of the *YMH* functional (1.1) are characterized by the solutions to the system:

$$(3.1) \quad D^*F - {}^*J = 0,$$

$$(3.2) \quad {}^*D^*D\phi + (\lambda|\phi|^2 - m^2)\phi = 0,$$

where the components of the current  $J$  are given by:

$$(3.3) \quad J^\alpha := -(D\phi, \rho_*(X_\alpha)\phi) ; \quad X_\alpha \in \mathcal{SU}(2) ,$$

with the matrix inner product defined by  $(M, N) = \tilde{M} \cdot N$ , and the covariant derivative  $D\phi$  defined as

$$(3.4) \quad D\phi = d\phi + A^\alpha \rho_*(X_\alpha)\phi$$

(we recover the vector bundle  $E$  from the principal fiber bundle by using the representation  $\rho$  of the Lie group of  $SU(2)$  on  $\mathbf{C}^2 = \mathbf{R}^4$ ).

In local coordinates (3.1) and (3.2) may be written as :

$$(3.5) \quad D_j F^{\alpha j k} = -(\widetilde{D^k \phi}) \rho_*(e_\alpha)\phi,$$

$$(3.6) \quad D_i D^i \phi = (\lambda|\phi|^2 - m^2)\phi,$$

where the symbol  $\sim$  in (3.5) denotes the transposition on column vectors. We now introduce the  $T^4$ -invariant pair given by (2.2) and (2.5-8). First define the vectors

$$(3.7) \quad A = \left(\frac{a_1}{r_1}, \dots, \frac{a_4}{r_4}\right), B = \left(\frac{b_1}{r_1}, \dots, \frac{b_4}{r_4}\right), C = \left(\frac{c_1}{r_1}, \dots, \frac{c_4}{r_4}\right),$$

and their products

$$(3.8) \quad A \cdot B = \sum (a_i b_i / r_i^2),$$

similarly for  $B \cdot C$ ,  $A^2 = A \cdot A$ , etc. Also for the parameters in (2.5) define the quantity

$$(3.9) \quad K = x_1^2 + x_2^2 + x_3^2 + x_4^2.$$

From the evaluation of (3.5) in a  $T^4$ -invariant pair we get

$$(3.10) \quad b_i(A \cdot B) + c_i(A \cdot C) - a_i(B^2 + C^2 - K/4) = -K/4,$$

$$(3.11) \quad a_i(A \cdot B) + c_i(B \cdot C) - b_i(A^2 + C^2 - K/4) = 0,$$

$$(3.12) \quad a_i(A \cdot C) + b_i(B \cdot C) - c_i(A^2 + B^2 - K/4) = 0,$$

while evaluation of (3.6) results in

$$(3.13) \quad R + \frac{1}{2} \left( \frac{a_1}{r_1^2} + \frac{a_2}{r_2^2} + \frac{a_3}{r_3^2} + \frac{a_4}{r_4^2} \right) + \frac{1}{4} (A^2 + B^2 + C^2) + \lambda K - m^2 = 0,$$

where by definition

$$(3.14) \quad R = \frac{1}{4} \left( \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r_4^2} \right).$$

Equations (3.10–3.13) describe the coupling between  $\Lambda$  and  $\phi$ .

*Proof of Theorem 1.* We proceed by construction. First in order to simplify the above set of equations we choose  $A, B, C$  such that:

$$(3.15) \quad A \cdot B = A \cdot C = B \cdot C = 0.$$

Furthermore, it is easy to verify that in this case (3.11) and (3.12) imply,

$$(3.16) \quad C^2 = B^2, \quad A^2 + C^2 = \frac{K}{4}.$$

Hence a particular solution to (3.10) is

$$(3.17) \quad a_i = K/(8C^2 - K).$$

Moreover, using (3.17) in equations (3.15) implies

$$(3.18) \quad \frac{b_1}{r_1^2} + \cdots + \frac{b_4}{r_4^2} = 0, \quad \text{and} \quad \frac{c_1}{r_1^2} + \cdots + \frac{c_4}{r_4^2} = 0,$$

and substitution of (3.15) in (3.16) yields

$$(3.19) \quad \left( \frac{K}{8C^2 - K} \right)^2 \left( \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} + \frac{1}{r_4^2} \right) = \frac{K}{4} - C^2.$$

If we now define

$$(3.20) \quad 4C^2 = X,$$

we can rewrite equation (3.19) as

$$(3.21) \quad X^3 - 2KX^2 + (5/4X + 4R)K^2 - K^3/4 = 0.$$

An analysis of (3.20) shows that:

If  $16R \leq K < 216R$ , then there exists only one real positive root.

If  $K = 216R$ , then there are three real equal roots.

If  $K > 216R$ , then there are three real different roots.

We shall exclude from consideration the case  $K = 16R$ , since this correspond to  $X = 0$  which implies, in turn that  $a_i = -1$ ,  $b_i = c_i = 0$ . For such values the curvature is zero and thus the Yang-Mills part of the action functional becomes a pure gauge. Note in particular that the value of  $R$  determines the minimum value of  $K$  for which there are solutions.

An additional straightforward computation shows that these solutions to the cubic (3.20), with  $K > 16R$ , together with

$$(3.22) \quad A = \frac{K}{2X - K} \left( \frac{1}{r_1}, \frac{1}{r_2}, \frac{1}{r_3}, \frac{1}{r_4} \right),$$

$$(3.23) \quad B = \frac{1}{4} \frac{X}{R} \left( \frac{1}{r_2}, -\frac{1}{r_1}, -\frac{1}{r_4}, \frac{1}{r_3} \right),$$

$$(3.24) \quad C = \frac{1}{4} \frac{X}{R} \left( \frac{1}{r_3}, \frac{1}{r_4}, -\frac{1}{r_1}, -\frac{1}{r_2} \right),$$

are appropriate non-trivial coupled solutions to the inhomogeneous field equation (3.1).

Finally substituting (3.17) and (3.22-3.24) in (3.13) and making repeated use of (3.19) yields

$$(3.25) \quad \left( \sqrt{R} + \frac{\sqrt{K - X}}{4} \right)^2 + \frac{X}{8} = m^2 - \lambda K.$$

Thus for a given  $K$ , (3.25) relates the mass and  $\lambda$  parameters. For an admissible value of  $\lambda$ , the above values of  $\Lambda$ , and  $K$  (which determines  $\phi$ ), give solutions to (3.1-3.2).

Observe that since the vector bundle  $E$  is trivial, the minimum value of the  $YMH$  functional is zero. However a direct computation shows that

the value of the functional for the above solutions is greater than zero, hence these are non-minimal. This ends the proof of Theorem 1.  $\square$

*Remark 2.* Note also that since the left side of (3.25) is positive definite, and since  $K > 16R$ , we must have

$$(3.26) \quad 16\lambda R \leq m^2$$

which sets a lower bound for the mass of the Higgs field.

*Remark 3.* The solutions found here can not be predicted by Theorem 3.1 in [10], this follows from the fact that the space of  $T^4$ -invariant pairs  $\{(\Lambda, \phi)\}$  contains reducible connections (the family of solutions includes flat connections), hence the space  $\{(\Lambda, \phi)\}$  modulus gauge equivalence is not a smooth manifold. In addition we do not use the same lift of the  $T^4$ -action to define both  $\Lambda$  and  $\phi$ , which is also required to arrive at the results in [10].

#### 4. Gauge equivalence.

Recall that we have been working on a finite dimensional slice  $\{g, (\Lambda, \phi)\}$  of the full space of metrics and fields, so have the action of two groups of transformations: The group of diffeomorphisms of  $T^4$  acting on the metrics and the group of gauge transformations. As shown by Parker [P1] the *YMH* functional is constant along conformal orbits of  $\{g\}$ , and on gauge orbits of  $\{(\Lambda, \phi)\}$ , so the solutions in Section 3 will be classified up to equivalence  $\sim$  under the above groups. The global structure of the quotient space  $\{g, (\Lambda, \phi)\} / \sim$  is very complicated because there are many points with non-trivial isotropy. However the construction of local sections is possible. In fact the global conformal structures of  $\{(T^4, g)\}$  are easy to describe. For example they can be determined by fixing the parameter  $r_4 = 1$  and leaving  $\{r_1, r_2, r_3\}$  free in (2.1).

On the other hand the action of the gauge group can be used to classify the invariant connections in (2.2) into classes modulo gauge transformations. For this purpose note first that by Theorem 11.8 in [7] the holonomy group  $Hol_\theta(\Lambda)$  associated to the connection given by  $\Lambda$ , which is generated by

$$(4.1) \quad m_0 + [\Lambda(X_n), m_0] + \dots,$$

where  $m_0 \in SU(2)$  is generated by

$$(4.2) \quad \{[\Lambda(X_n), \Lambda(X_m)] - \Lambda([X_n, X_m]) | X_m, X_n \in \text{Lie algebra of } (T^4)\}.$$



If  $\Lambda$  is not a Lie algebra homomorphism and is given by a matrix of rank three, then its holonomy group is the whole of  $SU(2)$ . Consequently the centralizer of the holonomy group is the identity, so these connections are generic. It is easy to show that the above conditions define an open subset in the space of all  $\{\Lambda\}$ , hence genericity is also an open condition in the space of  $T^4$ -invariant connections.

We now describe some families of  $T^4$ -invariant connections which are transverse to the action of the gauge group by making use of Proposition 4.12 in [1], which for our present case reduces to:

**Proposition 4.** [1] *Let  $\Lambda_1$  and  $\Lambda_2$  be two generic  $T^4$ -invariant connections. The connections are gauge equivalent iff there exists a  $u \in SU(2)$  such that*

$$(4.3) \quad \Lambda_2 = \text{ad}(u^{-1})\Lambda_1,$$

where, after expressing  $u$  in terms of the Euler angles  $\{(\beta, \gamma, \psi)\}$ , the adjoint is given by

$$(4.4) \quad \text{ad}(u^{-1}) = \begin{pmatrix} \cos \psi \cos \beta - \sin \psi \sin \beta \cos \gamma & \cos \psi \sin \beta + \sin \psi \cos \beta \cos \gamma & \sin \gamma \sin \psi \\ -\sin \psi \cos \beta - \cos \psi \sin \beta \cos \gamma & \cos \psi \cos \beta \cos \gamma - \sin \psi \sin \beta & \sin \gamma \cos \psi \\ \sin \gamma \sin \beta & -\sin \gamma \cos \beta & \cos \gamma \end{pmatrix}$$

Note that (4.4) acts on the vectors columns of (2.2) by a Euclidean rotation.

Using the above it becomes possible to put three elements in any matrix  $\Lambda$  equal to zero, arranged so that two can be in any chosen column and the third in any entry of the remaining ones. Thus, for instance, taking

$$(4.5) \quad \begin{aligned} \cos \gamma &= \frac{b_1 \cos \beta - a_1 \sin \beta}{[(b_1 \cos \beta - a_1 \sin \beta)^2 + c_1^2]^{1/2}}, \\ \cos \psi &= \frac{a_1 \cos \beta + b_1 \sin \beta}{(a_1^2 + b_1^2 + c_1^2)^{1/2}}, \\ \tan \beta &= \frac{b_2 c_1 - c_2 b_1}{a_2 c_1 - c_2 a_1}, \end{aligned}$$

we obtain

$$(4.6) \quad \Lambda_2 = \begin{pmatrix} a'_1 & a'_2 & a'_3 & a'_4 \\ 0 & b'_2 & b'_3 & b'_4 \\ 0 & 0 & c'_3 & c'_4 \end{pmatrix},$$

where the new parameters are determined in terms of the original ones by means of (4.3), (4.4) and (4.5).

The matrix (4.6) is a member of a 9-parameter family of  $T^4$ -invariant connections of the type described above which can be generated from (4.6) by additional gauge transformations with (4.4).

Another interesting family of 2-parameter generic connections arises when, in solving (3.10-12), we choose

$$(4.7) \quad \Lambda = \begin{pmatrix} a & a & a & a \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -c & c \end{pmatrix}.$$

It is straightforward to show that this family is not gauge related to the previous one. We get

$$(4.8) \quad 4a = \sqrt{K/R}, \quad c = r_3 \sqrt{K/8 + \sqrt{KR}/2}, \quad r_3 = r_4.$$

In addition note that substituting (4.8) in (3.13) yields

$$(4.9) \quad R + \frac{1}{8}\sqrt{K}(\sqrt{K} + 6\sqrt{R}) + \lambda K - m^2 = 0,$$

which clearly also leads to a lower bound for the mass parameter.

As a final remark we note that by following the procedure described above it is possible to complete the classification of the solutions to (3.10-12) into non-gauge equivalent classes with nine or less non-zero parameters. This is however beyond the scope of the present work.

## 5. Comments about stability.

There are many important results over compact and non-compact manifolds, where the stability of the Yang-Mills or Yang-Mills-Higgs fields imply the non-existence or the triviality of the fields (see e. g. [12]). Hence it is interesting to study the stability for the solutions found in the preceding sections.

**Proposition 5.** *There are values of  $\lambda$  for which the family described in (4.8) admits some values of  $\phi$  and  $K$  such that they represent non-stable YMH fields.*

*Proof.* To exhibit instability it is necessary that the second variation of the YMH action is negative. Thus as a first step, we would need to diagonalize the Hessian matrix of  $YMH(A, \phi)$ , restricted to the finite dimensional

space of invariant pairs  $\{\Lambda, \phi\}$ , at the critical points given by the solutions (3.22-25) and (4.8). Then applying the Morse Lemma for non-degenerate critical points, or the Gromoll-Meyer Splitting Lemma for degenerate critical points, we could derive germs of the  $YMH(A, \phi)$  at the points which would exhibit the geometric behavior of the associated quadric from its signature. For our purposes, however, it is simpler to consider the determinant of the Hessian and show that it is negative for some values of the parameter  $K$  which characterizes the field  $\phi$ . Again our proof is based on construction, and in Fig. 1 we display a three dimensional graph of the determinant of the Hessian obtained by means of a computer calculation for (4.8) and with the solution parameters varying over the values  $K \in (0, 20)$  and  $\lambda \in (0, 20)$ .

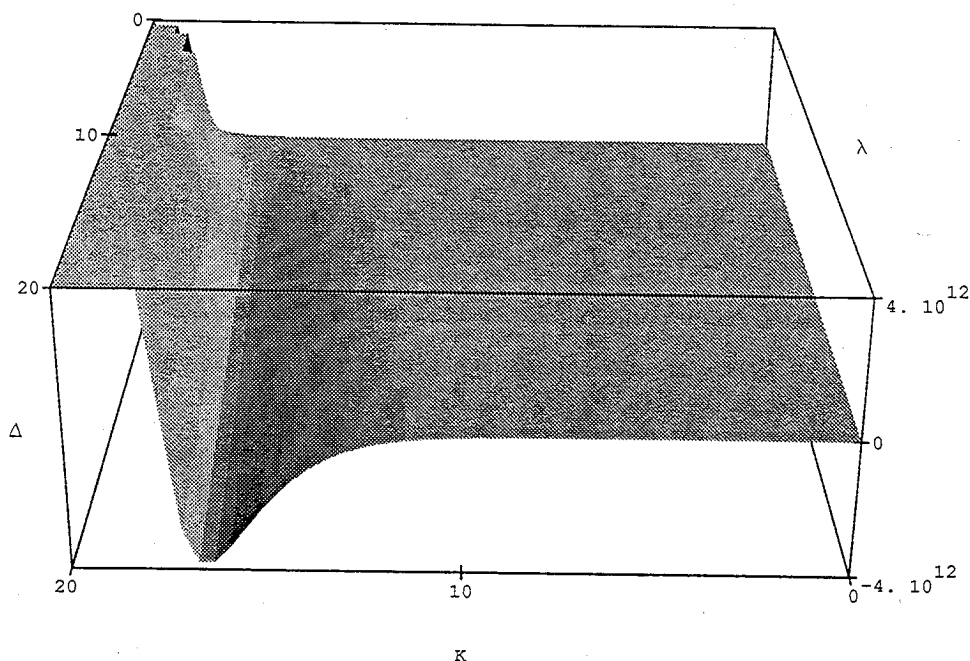


Figure 1 The determinant  $\Delta$  of the Hessian for the  $YMH$ -action density vs.  $\lambda$  and  $K$  for the solutions (4.7) with  $r_i = 1$ .

This graph clearly shows the existence of negative values for some  $\lambda$  and  $K$ . Hence the instability of the functional over the whole space of fields follows.  $\square$

Note in fact that a continuous family of non-stable *YMH* fields arises from the above numerical computation, since *YMH* fields  $(\Lambda, \phi)$  and  $(\Lambda', \phi')$  which correspond to different values of  $K$ , for which the determinant is negative, are not gauge equivalent. Obviously the variations of  $\lambda$  determine quantitatively different *YMH* functionals.

For the family of *YMH* fields given by (3.22–25), a similar numerical computation yields an always positive determinant for the finite dimensional Hessian. Thus, in order to complete the study of stability in this case, one would need to follow the procedure of finding the full set of eigenvalues of the Hessian for these solutions. The main difficulty in completing this program resides in the complex dependence of these eigenvalues on the parameters  $\lambda$  and  $K$ , which we consider somewhat beyond the scope of the present work.

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