Looking Back on Gauss and Gaussian Legends: Answers to the Quiz from 37(4)

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Years Ago *features essays by historians and mathematicians that take us back in time. Whether addressing special topics or general trends, individual mathematicians or "schools" (as in schools of fish), the idea is always the same: to shed new light on the mathematics of the past. Submissions are welcome.*

➢ Submissions should be uploaded to http://tmin.edmgr.com or sent directly to **David E. Rowe,** e-mail: rowe@mathematik.uni-mainz.de

In the December 2015 issue of this column, I posed eight questions about the near-legendary Carl Friedrich Gauss (1777–1855). When dealing with someone as famous as Gauss, it certainly isn't always easy to sort out fact from fiction, but trying to do so can be instructive. So this quiz began with a legend of rather recent vintage. It concerns a certain episode told by Daniel Kehlmann in his entertaining historical novel, *Measuring the World* (Kehlmann 2006). For those unfamiliar with this book, or the movie based on it, just a word about the title. The story begins with Gauss travelling to Berlin to meet the naturalist Alexander von Humboldt. Kehlmann portrays these two heroes of German science as oddball types; for some reason, both of them are obsessed by a need to measure terrestrial things. Beyond that realm, however, Gauss even reflects on certain new possibilities for measuring space itself.

Several of the queries in the quiz touch on Gauss's interest in testing non-Euclidean geometry, beginning with Kehlmann's new approach to this theme. From Gauss's correspondence with friends and colleagues, it has long been known that he had seriously contemplated an alternative to Euclidean geometry. But he also hated public controversy, which probably accounts for why he remained silent, even when others (notably N. Lobachevsky and J. Bolyai) stuck their necks out. Gauss was, in this respect, conservative to the core.

Kehlmann puts a new spin on all this, though, by staging a fictitious meeting between Gauss and an aged philosopher who happened to live in far off Königsberg: an old sage by the name of Immanuel Kant. Picking up with this episode in the novel, we learn that the young Gauss has recently stumbled on the thought that Euclidean geometry might not be true if we measured the world more carefully. Knowing that the great Kant had taught that the theorems of Euclidean geometry are transcendent truths that govern our perceptions of space, and which are therefore independent of all possible measurement results, he sets off for Königsberg hoping to attain enlightenment. Since Gauss was living in Brunswick, this journey of nearly 1000 kilometers would in any event have been an arduous undertaking, lasting perhaps two weeks (one way!). Here Kehlmann's omniscience helps us appreciate Gauss's mental state on arrival:

> When he reached Königsberg Gauss was almost out of his mind with exhaustion, back pain, and boredom. He had no money for an inn, so he went straight to the university and got directions from a stupid-looking porter. Like everyone here, the man spoke a peculiar dialect, the streets looked foreign, the shops had signs that were incomprehensible, and the food in the taverns didn't smell like food. He had never been so far from home.

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DOI 10.1007/s00283-016-9662-1

Gauss had some difficulty persuading Kant's butler that he had come to consult with the master about a matter of great urgency. Whether persuaded by this or not, the butler reluctantly gave in and escorted this persistent visitor into Kant's private chambers. At this climactic point, the great chance he was waiting for, Gauss could now finally pour his heart out, hoping to thereby gain the wise counsel that brought him all this way:

> In a hushed voice, he made his request: he had ideas he had never been able to share with anyone. For example, it seemed to him that Euclidean space did not, as per the *Critique of Pure Reason,* dictate the form of our perceptions and thus of all possible varieties of experience, but was, rather, a fiction, a beautiful dream. The truth was extremely strange: the proposition that two given parallel lines never touched each other had never been provable, not by Euclid, not by anyone else. But it wasn't at all obvious, as everyone had always assumed. He, Gauss, was thinking that the proposition was false. Perhaps there were no such things as parallels. Perhaps space also made it possible, provided one had a line and a point next to it, to draw infinite numbers of different parallels through this one point. Only one thing was certain: space was folded, bent, and extremely strange.

Many readers will know what happens next, but for those who don't, I'll say no more. Overlooking all the mathematical obscurities in this passage—Kehlmann is just telling a story after all—I raised question one: how do we know that this meeting never took place?

Of course actually proving that a certain event in the distant past did not happen or could not have occurred is next to impossible, so certainly we should not neglect considerations of plausibility here. For example, I am unaware of any evidence that Gauss ever expressed a positive view regarding Kant's famous doctrine regarding the geometry of space. On the contrary, he seems to have held a skeptical view of Kantian epistemology in general. This being the case, to suspend our disbelief would presumably mean swallowing the idea that even Gauss was impressed by Kant's reputation. If so, he might well have been eager to learn his reaction when confronted with a sharply different understanding of the theory of parallels. Let us, then, allow for this possibility.

We can next turn to certain circumstantial and biographical details; these, at any rate, were the type of data I had in mind when I first thought of this question. First, we should note that Kant died on 12 February 1804, which means that Gauss would have had to visit him by early 1804, at the very latest. Thankfully, Kehlmann provides us with just enough information so that we can quite easily zero in on the time of this alleged journey. That's because his Gauss is not only in a desperate state of mind owing to his dark thoughts regarding the strange non-Euclidean character of space. He is probably even more distraught because he has recently fallen madly in love with a lovely young girl named Johanna Osthoff. Like him, she is a Brunswick native from a similar working-class family background. When he proposes to her, though, she rebuffs him with worries about their compatibility. Soon after this,

Figure 1. Lithograph of Gauss made by Siegfried Detlev Bendixen, published in *Astronomische Nachrichten* in 1828, the year Gauss met A. v. Humboldt in Berlin.

he suddenly decides to set off on his eastward journey in hopes of gaining reassurance from the elderly Kant.

This added romantic dimension works very nicely in the novel, and what is more, Kehlmann only needed to stretch the truth a tiny bit when describing the situation as regards his hero's romantic life. Little mystery remains on that score, for the extant letters from this time clearly show that Gauss was head over heels in love with Hannchen Osthoff, whom he eventually took as his wife on 9 October 1805. We also know that he first met her in July 1803. This, of course, left Kehlmann rather little time to conjure up the wedding proposal, which she then casts aside, prompting the heartbroken Gauss to rush off to Königsberg, and have him arrive there while Kant was still among the living. After all, he will need two weeks just to make the journey.

But even if this were all theoretically possible, it clearly did not happen because we know that the courtship only moved into high gear after Kant was already dead. This can be discerned from a letter Gauss wrote on 28 June 1804 to Farkas Bolyai, his Hungarian friend from their student days in Göttingen. There he expresses his deep feelings for Johanna, but he also confides that he had only recently dared approach her. Clearly dazzled by her beauty and character, he waited another month before proposing to her in writing. She accepted, and their engagement was announced on 22 November 1804. These letters, together with many more details, can be found in Biermann's work (1990). Gauss's correspondence travelled far and wide—some 7000 of his letters are still extant—but he himself never set foot in Königsberg.

The second query concerned one of the best-known stories found in virtually every biography of Gauss. As a youngster he stunned his grade-school teacher by quickly adding up a series of natural numbers almost before the other little boys had a chance to begin calculating. The source for this tale was actually Gauss himself, as we learn from Wolfgang Sartorius von Waltershausen, his friend and first biographer:

> In 1784, after his seventh birthday, the little fellow entered the public school, where elementary subjects were taught, and which was under the supervision of a man named Büttner… Here occurred an incident which [Gauss] often related in old age with amusement and relish. In this class the pupil who first finished his example in arithmetic was to place his slate in the middle of a large table. On top of this, the second placed his slate, and so on. The young Gauss had just entered the class when Büttner gave out a problem for summing an arithmetic series. The problem was barely stated before Gauss threw his slate on the table with the words (in the low Brunswick dialect): "There it lies." While the other pupils continued counting, multiplying, and adding, Büttner, with self-conscious dignity, walked back and forth, occasionally throwing an ironical, pitying glance toward the youngest of the pupils. The boy sat quietly with his task ended, as fully aware as he always was on finishing a task that the problem had been correctly solved and that there could be no other result. At the end of the hour the slates were turned bottom up. That of the young Gauss with one solitary figure lay on top. When Büttner read out the answer, to the surprise of all present that of young Gauss was found to be correct, whereas many of the others were wrong (Satorius 1856, 12).

This is surely the earliest written account of this famous story, but I noted that here, as well as in other older sources, one finds no mention of the actual problem Gauss solved so quickly, whereas later versions typically refer to the numbers $1 + \ldots + 100$. This led me to ask about the evidence underlying either version of this tale.

When I wrote this a little more than a year ago, I thought this would be a novel question to ask. But since then I have come to learn that others have asked this same question before. For example, it is taken up in *The Tower of Hanoi—Myths and Maths* (Hinz, et al. 2013, 35–36). My eyes were really opened, though, by a letter from José Hernández Santiago from Morelia, Mexico. Hernández informed me that Brian Hayes, a senior author for *American Scientist*, had published a thorough search of the literature, citing more than one hundred versions of this story. His results can be found online by going to: http://bit-player.org/wp-content/extras/gaussfiles/gauss-snippets.html. In his published article, "Gauss's Day of Reckoning" (Hayes 2006), he wrote that:

Figure 2. The house at Wilhelmstraße 30 in Brunswick where Gauss was born on 30 April 1777 (Photograph from Cajori [1912]).

> [the] *locus classicus* of the Gauss schoolroom story is a memorial volume published in 1856, just a year after Gauss's death. The author was Wolfgang Sartorius, Baron von Waltershausen, professor of mineralogy and geology at the University of Göttingen, where Gauss spent his entire academic career. As befits a funerary tribute, it is affectionate and laudatory throughout.

Hayes, of course, noticed the discrepancy I asked about, and he seemed to confirm my guess as to where the series came from. Regarding this he wrote: "In the literature I have surveyed, the 1–100 series makes its first appearance in 1938, some 80 years after Sartorius wrote his memoir. The 1–100 example is introduced in a biography of Gauss by Ludwig Bieberbach (a mathematician notorious as the principal instrument of Nazi anti-Semitism in the German mathematical community)."

Hernández informed me, though, that this book (Bieberbach 1938) was not the first place that refers to these numbers. In fact, Hayes has updated his list since its first appearance (2006), so it now contains an obscure pamphlet written in 1906 by one Franz Mathé. There one finds again that the task was to add the first hundred natural numbers. So perhaps Bieberbach took his version of the story from this source, or maybe an even older one. Hayes considered the possibility that: "someone to whom Gauss told the story 'with amusement and relish' [may have] left a record of the occasion. The existence of such a corroborating document cannot be ruled out, but at present there is no evidence for it… If an account from Gauss's lifetime exists, it remains so obscure that it can't have had much influence on other tellers of the tale" (Hayes 2006).

Sartorius von Waltershausen's memorial for Gauss (Sartorius 1856) has served as the principal source for numerous stories relating to the life of his friend (Reich 2012). Given their special relationship, there is good reason to view the information he related as highly reliable, and I exploited it for some of the quiz questions, including the third one. This concerns Gauss's aesthetic views regarding mathematical works, in which I asked about the source for his remark that the observer of an impressive building would surely wish to see it with the scaffolding removed. One finds this on page 82 of Sartorius (1856), followed by a statement that Gauss had come to prize this synthetic style of presentation through his studies of the works of Archimedes and Newton. Sartorius went on to explain that it was these sensibilities that informed the famous Gaussian motto "pauca sed matura" ("few but ripe").

Many of Gauss's readers took less delight in his Spartan writing style, which made it very difficult to penetrate the original line of thought that motivated the finished works. If we are to believe what one finds on Wikipedia, it would seem that Niels Henrik Abel was one of those who expressed such displeasure. Referring to Gauss's writing style, Abel supposedly said that: "He is like the fox who effaces his tracks in the sand with his tail." Since no reliable source for this statement is given, however, my fourth question asked whether this saying was due to Abel or perhaps someone else, and to determine on what occasion the remark was made.

It seemed to me quite odd that such a striking and colorful statement is nowhere to be found in the modern biographical literature, for example, in Ore (2008) or Stubhaug (2000). So I took the opportunity to pursue this question a few years ago in Kristiansand, Norway, not far from Abel's final resting place, during a visit with Reinhard Siegmund-Schultze. He happened to own the French translation of an older Abel biography by C. A. Bjerknes, and this source, indeed, contains a reference to the mysterious statement about Gauss as a fox, although only in a footnote (Bjerknes 1885, 92) and without attribution. Reinhard then suggested that we contact Henrik Kragh Sørensen, a leading expert on Abel. Henrik quickly solved the puzzle and sent us copies of two articles by Christopher Hansteen (1784–1873), published in March 1862 in the *Illustreret Nyhedsblad*. Hansteen was a well-known geophysicist, astronomer, and physicist in Christiania (present-day Oslo) who also happened to be Abel's mentor. His two essays were based on five letters he had received nearly 40 years earlier from Abel when the latter was touring Europe. The first of these five letters contains some interesting remarks apropos Gauss.

After tracking this down with the help of two historians, I was certain that this was one of the hardest of the eight questions in the quiz. So it was with great surprise that I learned from José Hernández that he had cracked this puzzle. He even sent me a scan of the page in *Illustreret Nyhedsblad* where one finds the first published reference to Gauss, the fox. Hernández notes that this magazine was a Norwegian weekly that was published in Christiania between 1851 and 1866. Among its contributors were such famous names in Norwegian literature as Ibsen, Bjørnson, and Collett. He also pointed out that passages from Abel's first letter to Hansteen can be found in Stubhaug's biography, where one reads:

> Abel [is remarking] on how all the young mathematicians in Berlin "nearly deify Gauss. He is for them the quintessence of all mathematical [e]xcellence." Abel went on to comment, "It may be granted that he is a great [g]enius, but it is also well-known that he gives rotten [l]ectures. Crelle says that all Gauss writes is an abomination [Gräuel], since it is obscure to the point of being almost impossible to understand" (Stubhaug 2000, 332).

It was in this connection that Hansteen added a footnote containing this statement:

A German student said about him on this occasion:
"er macht es wie der Fuchs, der wischt mit dem Schwanze seine Spuren im Sande aus" (Hansteen 2 March 1862).

Clearly, then, it was not Abel, but rather a German student whom he met in Berlin who suggested that the *princeps mathematicorum* resembled "a fox who effaces his tracks in the sand with his tail."

The fifth question concerned Gauss's attitude with regard to technological progress in general and the invention of the electromagnetic telegraph in particular. He and his collaborator, Wilhelm Weber, had come up with such a gadget in the early 1830s. Sartorius discusses various plans for its development in the Saxon railroad industry, but economic circumstances caused these to be dropped.

Illustreret Nyhedsblad.

Nr. 9. | Udkommer hver Søndag Morgen. | 2den Marts 1862. | Abonnementsprisen er 1 Spd. Halvaaret. | 11. Aarg.

Indhold.

Niels Henrik Abel. — Eventyr og Historier. — Brudstykker af Avisernes Historie. — Drammens Skolebygninger. — Korrespondance fra Paris. — Norge. — Blandede Efterretninger.

Niels Henrik Abel.

I.

Da denne senere som Mathematiker saa berømte unge Mand i 1821 var bleven akademisk Borger, viste han ved sin første Indtrædelse paa den akademiske Løbebane saa udmærkede Anlæg til de mathematiske Videnskaber, at flere af Universitetets Lærere, i Betragtning af hans trængende Omstændigheder, fandt sig opmuntrede til ved et Sammenskud at bidrage til, at han ved et forlænget Ophold ved Universitetet kunde erholde Leilighed til, end yderligere at dyrke en Videnskab, hvortil Naturen havde givet ham et saa umiskjendeligt Kald.

Efter to Aars Forløb behagede det hans Majestæt, efter Anbefaling af det akademiske Kollegium, at

rommelse, som vor Abel. Maaskee derfor Redaktionen vil forunde følgende 4 Breve, der skildre hans udmærkede Arbeidsdygtighed under Reisen, og hans elskværdige Charakteer, en Plads i Bladet.

Hansteen.

1.

Berlin, 5te December 1825.

Vel kunde og burde jeg maaskee før have skrevet Dem til Hr. Professor; men jeg ønskede først at være kommet i Orden med Et og Andet, saa jeg kunde have Anledning til at fortælle, hvorledes jeg har og vil have Nytte af mit Ophold her. De har maaskee forundret Dem over, hvorfor jeg først reiste til Tydskland; men dette gjorde jeg deels fordi jeg da kom til at leve sammen med Bekjendtere, deels fordi jeg da var mindre udsat for ikke at anvende Tiden paa bedste Maade; saasom jeg kan forlade Tydskland hvad Dieblik det skal være for at reise til Paris, som bør være det vigtigste Sted for mig.

Her i Berlin har jeg just ikke gjort betydelig

for Mathematiken, ligesom i Frankrig, sagde han, at han længe havde havt i Sinde at ville paatage sig Redaktionen af en saadan, og vilde ogsaa med det Første see at sætte det i Udøvelse. — Nu er den bragt i Stand, og det til min store Glæde; thi da har jeg et Sted, hvor jeg kan faae en eller anden af mine smaae Frembringelser trykt. Jeg har allerede udarbeidet 4, som kommer til at staae i det første Hefte; og da de ere skrevne paa Fransk, saa er Crelle saa galant, at oversætte dem. Min Smule Fransk kommer mig saaledes godt til Nytte. Crelle ytrede om Formen i mine Afhandlinger, at han syntes de vare meget tydeligt og godt skrevne, hvilket glæder mig særdeles, da jeg altid har været bange for, at jeg vilde have vanskeligt for at udvikle mine Tanker paa en ordentlig Maade. Dog raadede han mig til at være lidt vidtløftigere, især her i Tydskland. Han tilbød mig ogsaa Honorar for mine Afhandlinger, hvilket jeg naturligviis ikke havde gjort Regning paa, og som jeg ogsaa frabad mig; dog syntes jeg at mærke, at han

Figure 3. Christopher Hansteen's first article on Abel in *Illustreret Nyhedsblad* (Photograph by Stan Sherer).

Not until the invention of relays, however, was it possible to transmit messages over longer distances, a breakthrough exploited by Samuel F. B. Morse. Gauss expressed the view that science should by all means befriend the practical arts, but should never be a slave to the latter. Perhaps he said this often, but his former student Moritz Abraham Stern (1807–1894) recorded this statement for posterity in 1877, on the occasion of the Gauss centenary celebrations in Göttingen (Stern 1877, 15).

For many decades Stern was a fixture in the Göttingen mathematical community, having begun teaching there in 1830 as a Privatdozent. As an unbaptized Jew, many were convinced that he would have virtually no chance of becoming a full professor, although in 1848 he was finally appointed to an extraordinary professorship. Throughout these years he taught many subjects, but also as a researcher he was generally held in high esteem by his colleagues. Finally, on 30 July 1859, the Hannoverian Ministry of Culture granted him the title of Ordinarius, the very day that Bernhard Riemann was also appointed full professor.

We come now to the last three quiz questions, which revolve around the measurement of space, as discussed earlier. Kehlmann's Gauss was an astronomer and geodesist, befitting the title of the novel, but he also had the rich imagination of a troubled genius. So, like the real Gauss, he thought a lot about an ''anti-Euclidean'' geometry, and we know that the real Gauss wrote about this topic in letters to colleagues and friends. During the years 1818 to 1826 he was charged with surveying the Kingdom of Hanover, for which purpose he invented the heliotrope to improve the accuracy of sightings. The sixth question concerns a famous test of Euclidean geometry that Gauss was purported to have carried out based on measurements taken for a giant spherical triangle whose vertices were located in three remote mountainous locations: Brocken in the Harz, Hohen Hagen near Dransfeld south of Göttingen, and Inselberg in the Thuringian Forest. These three points happen to be separated by quite considerable distances: 69, 85, and 107 kilometers, respectively, but by using a heliotrope all three of these points could be sighted from Gauss's observatory in Göttingen.

As is now well known, Sartorius is the source for the story that Gauss used the data from these sightings to test whether the sum of the angles in this giant optical triangle deviated from 180°. The result he obtained was astonishingly small, only two-tenths of one second of arc from the Euclidean result (Sartorius 1856, 53). In describing Gauss's views on the foundations of geometry, Sartorius wrote:

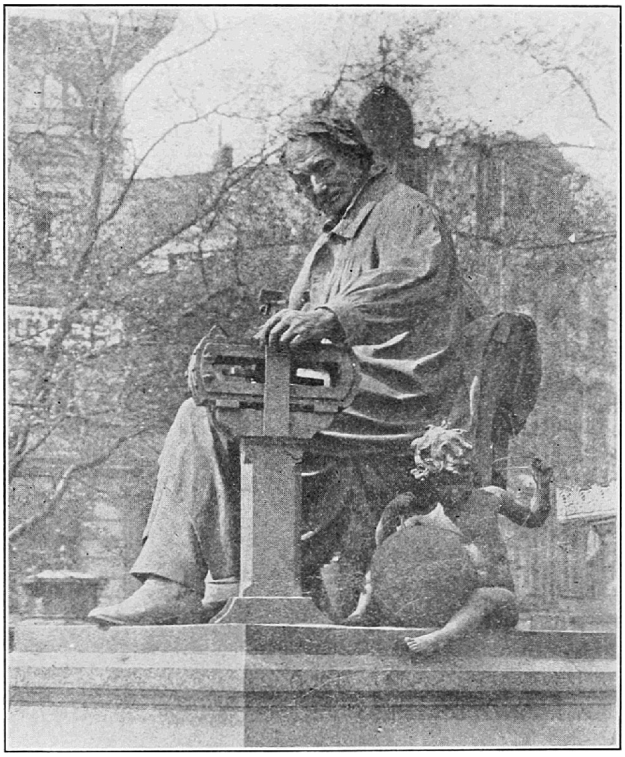

Figure 4. This statue of Gauss sending a message on his telegraph could once be seen at one corner of the Potsdam Bridge in Berlin-Schöneberg. The other three corners were adorned with statues of Röntgen, Siemens, and Helmholtz. All four were erected in 1898 and were destroyed during the bombing of Berlin in 1944 (Photograph from Cajori [1912]).

> Gauss regarded geometry as a consistent structure only if the theory of parallels is conceded as an axiom and placed at the summit. He was nevertheless convinced that this proposition could not be proved, even though one knew from experience, e.g. from the angles in the triangle Brocken, Hohen Hagen, and Inselberg that it was approximately correct (Sartorius 1856, 81).

This answers the first part of question six, but it hardly helps in answering the second, namely, how believable is it that Gauss used the BHI triangle to test the possibility that space might be curved? The historian of physics Arthur Miller disputed this claim in a short paper entitled "On the myth of Gauss's experiment regarding the Euclidean nature of space" (Miller 1972). Miller based his critique on arguments taken from Gauss's work on surface theory—in particular his discussion of the same triangle in *Disquisitiones generales circa superficies curvas* (1827)—but apparently he was unaware of Sartorius's testimony cited previously. Thus, he speculated that this "Gaussian myth" only arose in the wake of Einstein's gravitational theory, which drew interest to Riemann's famous lecture of 1854 on higher-dimensional manifolds with intrinsic curvature, a generalization of Gauss's 1827 theory, which dealt with the intrinsic curvature properties of surfaces embedded in Euclidean 3-space.

Miller's argument was immediately challenged by B. L. van der Waerden (van der Waerden 1974), but the first detailed examination of the issues at stake came a decade later with the publication of Breitenberger (1984). More recently, Erhard Scholz has parsed the various issues involved in this debate (Scholz 2004). The thrust of his argument aims to show that Gauss had all the means at hand to calculate a deviation from 180° due to an assumption that the light rays in space followed geodesics in a spatial geometry of constant curvature.

With this in mind, the seventh question about the results Gauss would have obtained based on his own data can best be answered by referring readers to the detailed analysis in Scholz (2004). In fact, Breitenberger's paper also confirms that the figure given by Sartorius accords well with Gauss's data, but he argues that Gauss lacked a clear conception of a 3-dimensional non-Euclidean geometry within which he could frame an alternative to the Euclidean theory (Breitenberger 1984, 285). Scholz, on the other hand, cites Gauss's letter to Schumacher from 1831, in which Gauss states that "we know from experiment that the constant k [in anti-Euclidean geometry] must be incredibly large compared with all that we can measure. In Euclid's geometry k is infinite" (Scholz 2004, 24). He thus finds the testimony of Sartorius fully believable and consistent with what Gauss must have found.

Turning the historiographic tables, then, he dismisses what he calls "Miller's myth." For another view, however, one should take into account what Jeremy Gray (2006) writes about this and other difficulties associated with Gauss's place in the history of non-Euclidean geometry. As he states at the outset: "[the evidence] suggests that Gauss was aware that much needed to be done to Euclid's Elements to make them rigorous, and that the geometrical nature of physical Space was regarded by Gauss as more and more likely to be an empirical matter, but in this his instincts and insights on this occasion were those of a scientist, not a mathematician" (Gray 2006, 60).

Kehlmann's myth only adds another layer of confusion to a story already shrouded in a good deal of mystery. In the meantime, his bestselling novel has given rise to a batch of efforts to separate fact from fiction in his various stories. Historians obviously have no such freedom; they are bound by documentary evidence. Miller's myth, as Scholz pointed out, was concocted without taking due account of a crucial piece of evidence that Sartorius obtained from Gauss himself. At the very least, Scholz's analysis (2004) clearly refutes Miller's claim that Gauss could not have tested the Euclidean hypothesis for spatial geometry.

The eighth and final question concerned the Gauss Tower, located on Hohen Hagen near Dransfeld. An announcement for it was made by Felix Klein and Karl Schwarzschild, who pointed to the significance of the location, which was meant to commemorate Gauss's purported test of non-Euclidean geometry. That same year David Hilbert invited a famous mathematician to attend the groundbreaking ceremony for its construction. This led me to ask: who was that mathematician and did he decide to accept Hilbert's invitation?

Figure 5. The Old Gauss Tower near Dransfeld (Dransfelder Archiv, http://dransfeld.knobelauflauf.de/).

This was a difficult question certainly, although Scott Walter has known the answer for a long time. He transcribed the letter Hilbert wrote to Poincaré on 25 February 1909 in which plans for Poincaré's Wolfskehl lectures, to be delivered in April, were discussed. This ends with these remarks: "…Finally, on the 30th of April, the birthday of Gauss, the dedication of a Gauss-Tower is planned. This will take place at nearby Dransfeld on the 'Hohenhagen' (a vertex of the Gaussian triangle of straight lines for which he observed that the angle sum was π). Your presence on that occasion is urgently wished." (The full letter in the original German can be found at http://henripoincarepapers.univ-lorraine.fr//chp/text/hilbert-1909-02-25.html.)

Surely Hilbert appreciated that Poincaré's own views on the nonempirical nature of this whole problem would have put him in an awkward position on such an occasion. For if Gauss really thought that his measurements of the great BHI triangle could possibly be used to test the empirical bounds within which Euclidean geometry held, then Poincaré would only be able to shake his head in disbelief, unless he wanted to explain to his hosts that even the great Gauss had not fully appreciated the nature of the situation. Luckily for Poincaré, the groundbreaking ceremony was postponed, allowing the distinguished French mathematician a chance to return to Paris in peace. The Gauss Tower was completed in 1911 and thereafter became a local tourist attraction. Basalt mining in the area surrounding its base caused it to collapse in 1963.

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