Vacuum State and the Propagator of the Scalar Field in 3+1 Sph-Sym LQG

Saeed Rastgoo[1]

CCM, UNAM

November 10, 2012

[1] In collaboration with Rodolfo Gambini and Jorge Pullin

Outline

1. The 3+1 Spherically Symmetric Model
   - The Hamiltonian
   - Quantization and the Master Constraint
   - States
   - The Propagator

Outline

1. The 3+1 Spherically Symmetric Model
   - The Hamiltonian
   - Quantization and the Master Constraint
   - States
   - The Propagator

Canonical Transformation: Generic to Standard 3+1

- Standard 3+1 Hamiltonian (in Ashtekar variables)

$$
\begin{aligned}
H =& N\Bigg(-\frac{E^\varphi}{2\sqrt{E^x}} - 2K_\varphi\sqrt{E^x}K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{E^x}} + \frac{((E^x)')^2}{8\sqrt{E^x}E^\varphi} - \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} \\
&+ \frac{\sqrt{E^x}(E^x)''}{2E^\varphi} + \frac{P_f^2}{2\sqrt{E^x}E^\varphi} + \frac{(E^x)^{3/2}(f')^2}{2E^\varphi}\Bigg) \\
&+ N^1\left((E^x)'K_x - E^\varphi(K_\varphi)' - P_f f'\right)
\end{aligned}
$$

is the sum of Hamiltonian and diffeomorphism constraints.

Canonical Transformation: Generic to Standard 3+1

- Standard 3+1 Hamiltonian (in Ashtekar variables)

$$
\begin{aligned}
H =& N\left(-\frac{E^\varphi}{2\sqrt{E^x}} - 2K_\varphi\sqrt{E^x}K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{E^x}} + \frac{((E^x)')^2}{8\sqrt{E^x}E^\varphi} - \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} \right.\\
&\left. + \frac{\sqrt{E^x}(E^x)''}{2E^\varphi} + \frac{P_f^2}{2\sqrt{E^x}E^\varphi} + \frac{(E^x)^{3/2}(f')^2}{2E^\varphi} \right)\\
&+ N^1\left((E^x)'K_x - E^\varphi(K_\varphi)' - P_f f'\right)
\end{aligned}
$$

is the sum of Hamiltonian and diffeomorphism constraints.

The 3+1 variables in (t, x, θ, ϕ) coordinates

- E^x and E^φ: related to radial and angular components of the densitized triad (basis vectors on the spatial hypersurfaces).

Canonical Transformation: Generic to Standard 3+1

- Standard 3+1 Hamiltonian (in Ashtekar variables)

$$\begin{aligned} H =& N\left(-\frac{E^\varphi}{2\sqrt{E^x}} - 2K_\varphi\sqrt{E^x}K_x - \frac{E^\varphi K_\varphi^2}{2\sqrt{E^x}} + \frac{((E^x)')^2}{8\sqrt{E^x}E^\varphi} - \frac{\sqrt{E^x}(E^x)'(E^\varphi)'}{2(E^\varphi)^2} \right. \\ & \left. + \frac{\sqrt{E^x}(E^x)''}{2E^\varphi} + \frac{P_f^2}{2\sqrt{E^x}E^\varphi} + \frac{(E^x)^{3/2}(f')^2}{2E^\varphi} \right) \\ & + N^1\left((E^x)'K_x - E^\varphi(K_\varphi)' - P_f f'\right) \end{aligned}$$

is the sum of Hamiltonian and diffeomorphism constraints.

The 3+1 variables in (t, x, θ, ϕ) coordinates

- E^x and E^φ: related to radial and angular components of the densitized triad (basis vectors on the spatial hypersurfaces).
- K_x and K_φ: related to radial and angular components of the extrinsic curvature of the spatial hypersurfaces.

Outline

1. The 3+1 Spherically Symmetric Model
   - The Hamiltonian
   - Quantization and the Master Constraint
   - States
   - The Propagator

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
    - **quantize** the model,

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,
  - study the **propagator of the scalar field on that state** .

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,
  - study the **propagator of the scalar field on that state** .

To choose a strategy consider an important problem...

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,
  - study the **propagator of the scalar field on that state** .

To choose a strategy consider an important problem...

- The **problem of dynamics**:

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,
  - study the **propagator of the scalar field on that state** .

To choose a strategy consider an important problem...

- The **problem of dynamics**:

Algebra of constraints in general relativity is a **non-Lie algebra**:

Strategy to Attack the Problem of Quantization

- Our aim in 3+1 case:
  - **quantize** the model,
  - propose and test a **vacuum state**,
  - study the **propagator of the scalar field on that state** .

To choose a strategy consider an important problem...

- The **problem of dynamics**:

Algebra of constraints in general relativity is a **non-Lie algebra**:

The Poisson bracket of Hamiltonian constraint with itself contains **structure functions** rather than constants.

Strategy to Attack the Problem of Quantization

The problem of dynamics

- The (smeared) Hamiltonian constraint is not a spatially diffeomorphism invariant function.

Strategy to Attack the Problem of Quantization

The problem of dynamics

- The (smeared) Hamiltonian constraint is not a spatially diffeomorphism invariant function.
- The algebra of (smeared) Hamiltonian constraints does not close, it is proportional to a spatial diffeomorphism constraint.

Strategy to Attack the Problem of Quantization

The problem of dynamics

- The (smeared) Hamiltonian constraint is not a spatially diffeomorphism invariant function.
- The algebra of (smeared) Hamiltonian constraints does not close, it is proportional to a spatial diffeomorphism constraint.
- The coefficient of proportionality is not a constant, it is a non-trivial function on the phase space.

$$\{\vec{C}(\vec{N}), \vec{C}(\vec{N}')\} = \kappa \vec{C}(\mathcal{L}_N \vec{N}')$$

$$\{\vec{C}(\vec{N}), C(N')\} = \kappa C(\mathcal{L}_N \vec{N}')$$

$$\{C(N), C(M)\} = \int d^3x (N\partial_a M - M\partial_a N) g^{ab} C_b$$

Strategy to Attack the Problem of Quantization

The master constraint

- The sum (smearing) of all the Hamiltonian constraints over space

$$\mathbb{H} = \int_{\Sigma} dx \frac{1}{2} \frac{[\mathcal{H}(x)]^2}{\sqrt{g}} \ell_{\mathrm{P}} = \int_{\Sigma} dx \frac{1}{2} \frac{[\mathcal{H}(x)]^2}{\sqrt{E^x(x) E^{\varphi}(x)}} \ell_{\mathrm{P}}$$

or its discrete version

$$\mathbb{H}^{\epsilon} = \sum_i \mathbb{H}(i) = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_{\mathrm{P}}}{\sqrt{g(i)}} = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_{\mathrm{P}}}{\sqrt{E^x(i) E^{\varphi}(i)}}$$

Strategy to Attack the Problem of Quantization

The master constraint

- The sum (smearing) of all the Hamiltonian constraints over space

$$\mathbb{H} = \int_{\Sigma} dx \frac{1}{2} \frac{[\mathcal{H}(x)]^2}{\sqrt{g}} \ell_P = \int_{\Sigma} dx \frac{1}{2} \frac{[\mathcal{H}(x)]^2}{\sqrt{E^x(x) E^\varphi(x)}} \ell_P$$

  or its discrete version

$$\mathbb{H}^\epsilon = \sum_i \mathbb{H}(i) = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{g(i)}} = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{E^x(i) E^\varphi(i)}}$$

- Claims that by using this:
  - Problems with the commuter algebra disappear.
  - Could have control of the solution space.
  - Could have control of the (quantum) Dirac observables of LQG.
  - Even a decision on whether the theory has the correct classical limit.
  - The connection with the path (or spin foam) formulation could be within reach.

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, make things simpler. Now left only with Hamiltonian constraint (one for each spatial point).

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, make things simpler. Now left only with Hamiltonian constraint (one for each spatial point).
  - **holonomize** the theory on a lattice.

Strategy to Attack the Problem of Quantization

- Configuration variable(s) are written as

$$K_\varphi \to \frac{\sin(\rho K_\varphi)}{\rho}$$

and one assumes that the radial direction is bounded with a spatial extent L and consists of discrete points x_i separated by a coordinate distance ϵ, thus e.g.

$$\int dx \to \epsilon \sum_x,$$

$$\delta(x-y) \to \frac{\delta_{x,y}}{\epsilon},$$

$$\frac{\delta}{\delta f(x)} \to \frac{1}{\epsilon}\frac{\partial}{\partial f},$$

$$f(x)' \to \frac{f(x_{i+1}) - f(x_i)}{\epsilon},$$

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, left only with Hamiltonian constraint (one for each spatial point).
  - **holonomize** the theory on a lattice.

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, left only with Hamiltonian constraint (one for each spatial point).
  - **holonomize** the theory on a lattice.
  - Introduce the **master constraint**:

$$\mathbb{H}^{\epsilon} = \sum_i \mathbb{H}(i) = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{g(i)}} = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{E^x(i) E^{\varphi}(i)}}$$

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, left only with Hamiltonian constraint (one for each spatial point).
  - **holonomize** the theory on a lattice.
  - Introduce the **master constraint**:

$$\mathbb{H}^{\epsilon} = \sum_i \mathbb{H}(i) = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{g(i)}} = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{E^x(i) E^{\varphi}(i)}}$$

  - **Physical states**: cylindrical functions in the **kernel of $\mathbb{H}$**, i.e. $\mathbb{H}|\Psi_{\text{cyl}}\rangle = 0$.

Strategy to Attack the Problem of Quantization

- Strategy to bypass the problem of dynamics:
  - **Partially fix the gauge**: eliminate the diffeomorphism constraint, left only with Hamiltonian constraint (one for each spatial point).
  - **holonomize** the theory on a lattice.
  - Introduce the **master constraint**:

$$\mathbb{H}^{\epsilon} = \sum_i \mathbb{H}(i) = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{g(i)}} = \sum_i \frac{1}{2} \frac{\mathcal{H}(i)^2 \ell_P}{\sqrt{E^x(i) E^{\varphi}(i)}}$$

  - **Physical states**: cylindrical functions in the **kernel of $\mathbb{H}$**, i.e. $\mathbb{H}|\Psi_{\text{cyl}}\rangle = 0$.
  - Find this kernel by variational technique: **minimize the expectation value of master constraint $\langle\Psi_{\text{cyl}}|\mathbb{H}|\Psi_{\text{cyl}}\rangle$**.

Quantized Master Constraint

- **Discretize** the Hamiltonian constraint $H = H_{\text{vac}} + GH_{\text{matt}}$, **holonomize the gravitational** degrees of freedom, **quantize the matter** degrees of freedom **using Fock** quantization (to be able to compare with QFT).

Quantized Master Constraint

- Partially gauge fixed ($E^x = x^2$), quantized Hamiltonian constraint reads

$$H(i) = -(1-2\Lambda)\epsilon - x(i+1)\frac{\sin^2(\rho K_\varphi(i+1))}{\rho^2} + x(i)\frac{\sin^2(\rho K_\varphi(i))}{\rho^2} + \frac{x(i+1)^3\epsilon^2}{(E^\varphi(i+1))^2} - \frac{x(i)^3\epsilon^2}{(E^\varphi(i))^2} + \ell_p^2\left(\frac{H^{(1)}_{\text{matt}}(i)}{(E^\varphi)^2(i)} + \frac{H^{(2)}_{\text{matt}}(i)\sin(\rho K_\varphi(i))}{\rho E^\varphi(i)} - H^{(3)}_{\text{matt}}(i)\right)$$

with $\Lambda = \frac{G}{2}\rho_{\text{vac}}$ and the vacuum energy density ρ_{vac} and

$$H^{(1)}_{\text{matt}}(i) = \left(\epsilon\left(P_f(i)\right)^2 + \epsilon x(i)^4\left(f(i+1) - f(i)\right)^2\right)\ell_P^2,$$
$$H^{(2)}_{\text{matt}}(i) = \left(-2x(i)\left(f(i+1) - f(i)\right)P_f(i)\right)\ell_P^2,$$
$$H^{(3)}_{\text{matt}}(i) = 2\rho_{\text{vac}}\epsilon\ell_P^2.$$

This way the expectation value of H_{matt} will be zero in the vacuum.

Quantized Master Constraint

- The discrete master constraint *operator* reads

$$\hat{\mathbb{H}}(i) = \ell_{\rm P} \Big[\hat{c}_{11}(i) \left(\hat{H}^{(1)}_{\rm matt}(i) \right)^2 + \hat{c}_{22}(i) \left(\hat{H}^{(2)}_{\rm matt}(i) \right)^2 + \hat{c}_{33}(i) \left(\hat{H}^{(3)}_{\rm matt}(i) \right)^2$$
$$+ \hat{c}_1(i) \hat{H}^{(1)}_{\rm matt}(i) + \hat{c}_2(i) \hat{H}^{(2)}_{\rm matt}(i) + \hat{c}_3(i) \hat{H}^{(1)}_{\rm matt}(i) + \hat{c}_{00}(i)$$
$$+ \hat{c}_{12}(i) \hat{H}^{(1)}_{\rm matt}(i) \hat{H}^{(2)}_{\rm matt}(i) + \hat{c}_{13}(i) \hat{H}^{(1)}_{\rm matt}(i) \hat{H}^{(3)}_{\rm matt}(i) + \hat{c}_{23}(i) \hat{H}^{(2)}_{\rm matt}(i) \hat{H}^{(3)}_{\rm matt}(i) \Big]$$

the $\hat{c}_i$ and $\hat{c}_{ij}$ coefficients contain only gravitational degrees of freedom, K_φ and E^φ.

Outline

1 The 3+1 Spherically Symmetric Model
- The Hamiltonian
- Quantization and the Master Constraint
- States
- The Propagator

Construction of the States: Gravitational Part

- We are interested in the vacuum solutions: classically correspond to $f = P_f = 0$. Thus for now we ignore H_{matt} and only consider H_{vac}:

$$H_{\text{vac}} = \left(-x(1 - 2\Lambda) - xK_\varphi^2 + \frac{x^3}{(E^\varphi)^2} \right)'$$

Construction of the States: Gravitational Part

- We are interested in the vacuum solutions: classically correspond to $f = P_f = 0$. Thus for now we ignore H_{matt} and only consider H_{vac}:

$$H_{\text{vac}} = \left(-x(1-2\Lambda) - xK_\varphi^2 + \frac{x^3}{(E^\varphi)^2} \right)'$$

- Make another gauge fixing $K_\varphi = 0$. Then weakly vanishing of H_{vac} means

$$E^\varphi = \frac{x}{\sqrt{1-2\Lambda}}$$

which is the classical solution.

Construction of the States: Gravitational Part

- Construct a polymer representation: set up a lattice of points $j = 0 \ldots N$ in the radial direction and write a "point holonomy" for the K_φ variable at each lattice site,

$$T_{\vec{\mu}} = \exp\left(i \sum_j \mu_j K_\varphi(j)\right) = \langle K_\varphi | \vec{\mu} \rangle$$

The quantities μ_i play the role of the "loop" in this one dimensional context.

Construction of the States: Gravitational Part

- The trial quantum states of the gravitational part $\langle\vec{\mu}|\psi_{\vec{\sigma}}\rangle$ are chosen to be centered around the classical solution

$$\langle\vec{\mu}|\psi_{\vec{\sigma}}\rangle = \prod_i \sqrt[4]{\frac{2}{\pi\sigma(i)}} \exp\left(-\frac{1}{\sigma(i)}\left(\mu_i - \frac{1}{\ell_{\mathrm{P}}^2}\left(\frac{\epsilon x(i)}{\sqrt{1-2\Lambda}}\right)\right)^2\right)$$

with the variable μ_i to be centered around the classical value of

$$E^{\varphi}(i) = \frac{\epsilon x(i)}{\sqrt{1-2\Lambda}}.$$

Construction of the States: Matter Part

- Find the expectation value of the matter Hamiltonian on the state $\langle\vec{\mu}|\psi_{\vec{\sigma}}\rangle$. This will be an operator $\hat{H}^{\text{eff}}_{\text{matt}}$ acting on the matter fields.

Construction of the States: Matter Part

- Find the expectation value of the matter Hamiltonian on the state $\langle\vec{\mu}|\psi_{\vec{\sigma}}\rangle$. This will be an operator $\hat{H}^{\text{eff}}_{\text{matt}}$ acting on the matter fields.
- Write $\hat{H}^{\text{eff}}_{\text{matt}}$ in terms of creation-annihilation $\hat{C}$ and $\hat{\bar{C}}$ operators by using Fourier analysis. It is

$$\hat{H}^{\text{eff}}_{\text{matt}} = \langle\psi_{\vec{\sigma}}|\hat{H}_{\text{matt}}|\psi_{\vec{\sigma}}\rangle = (1-2\Lambda)\int_0^{2\pi/\epsilon} d\omega\omega\hat{\bar{C}}(\omega)\hat{C}(\omega).$$

Construction of the States: Matter Part

- Find the expectation value of the matter Hamiltonian on the state $\langle\vec{\mu}|\psi_{\vec{\sigma}}\rangle$. This will be an operator $\hat{H}^{\text{eff}}_{\text{matt}}$ acting on the matter fields.
- Write $\hat{H}^{\text{eff}}_{\text{matt}}$ in terms of creation-annihilation $\hat{C}$ and $\hat{\bar{C}}$ operators by using Fourier analysis. It is

$$\hat{H}^{\text{eff}}_{\text{matt}} = \langle\psi_{\vec{\sigma}}|\hat{H}_{\text{matt}}|\psi_{\vec{\sigma}}\rangle = (1-2\Lambda)\int_0^{2\pi/\epsilon} d\omega\omega\hat{\bar{C}}(\omega)\hat{C}(\omega).$$

- The vacuum states $|0\rangle$ are the states of operator $\hat{H}^{\text{eff}}_{\text{matt}}$, annihilated by $\hat{C}$.

The Full Trial State

- The full trial states: direct product of the vacuum of the matter part of the Hamiltonian and the Gaussian on the gravitational variables:

$$|\psi_{\vec{\sigma}}^{\text{trial}}\rangle = |\psi_{\vec{\sigma}}\rangle \otimes |0\rangle$$

The Full Trial State

- The full trial states: direct product of the vacuum of the matter part of the Hamiltonian and the Gaussian on the gravitational variables:

$$|\psi_{\vec{\sigma}}^{\text{trial}}\rangle = |\psi_{\vec{\sigma}}\rangle \otimes |0\rangle$$

- To find the physical states, $\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle_{\text{phys}} = 0$, vary $\vec{\sigma}$ to find the minimum of the expectation value of the master constraint on the trial states $\langle\psi_{\vec{\sigma}}^{\text{trial}}|\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle$.

Minimizing the Master Constraint

- Minimizing $\langle\psi^{\text{trial}}_{\vec{\sigma}}|\hat{\mathbb{H}}|\psi^{\text{trial}}_{\vec{\sigma}}\rangle$ results in

$$\langle\psi^{\text{trial}}_{\vec{\sigma}}|\hat{\mathbb{H}}|\psi^{\text{trial}}_{\vec{\sigma}}\rangle = \frac{\sigma_0 \ell_{\text{P}}^3}{\epsilon x^2} + \mathcal{O}(\ell_{\text{P}}^5)$$

Minimizing the Master Constraint

- Minimizing $\langle\psi_{\vec{\sigma}}^{\text{trial}}|\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle$ results in

$$\langle\psi_{\vec{\sigma}}^{\text{trial}}|\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle = \frac{\sigma_0 \ell_{\mathrm{P}}^3}{\epsilon x^2} + \mathcal{O}(\ell_{\mathrm{P}}^5)$$

  - we have assumed

$$\sigma = \sigma_0 \frac{\epsilon^2}{\ell_{\mathrm{P}}^2}$$

with σ_0 of order unity.

Minimizing the Master Constraint

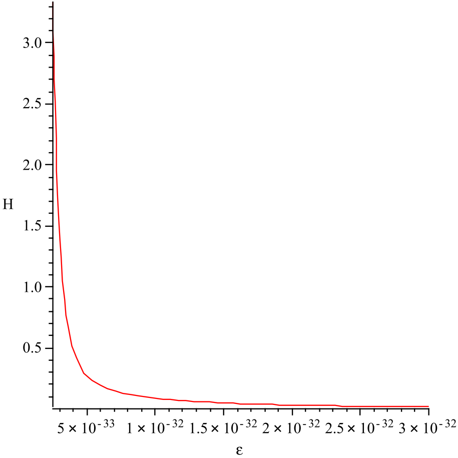
- Minimizing $\langle\psi_{\vec{\sigma}}^{\text{trial}}|\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle$ results in

$$\langle\psi_{\vec{\sigma}}^{\text{trial}}|\hat{\mathbb{H}}|\psi_{\vec{\sigma}}^{\text{trial}}\rangle = \frac{\sigma_0 \ell_{\text{P}}^3}{\epsilon x^2} + \mathcal{O}(\ell_{\text{P}}^5)$$

  - we have assumed
  $$\sigma = \sigma_0 \frac{\epsilon^2}{\ell_{\text{P}}^2}$$
  with σ_0 of order unity.
  - Also we have assumed σ independent of x. Our experiments suggest that allowing variations in x leads to the same minimum value of σ approximately independent of x.

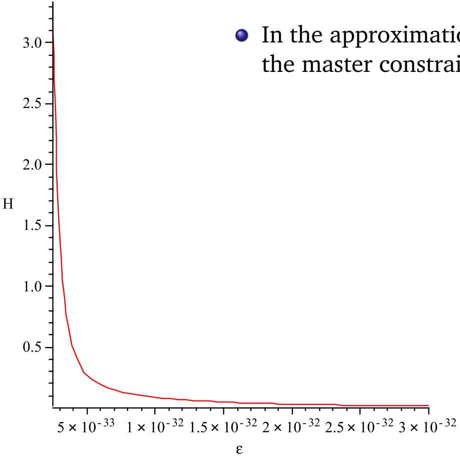
Minimizing the Master Constraint

Study of the $\langle\hat{\mathbb{H}}\rangle_{\min}$ for different choices of $\epsilon\ (\ell_p)$ and different powers of ϵ/ℓ_P :



Minimizing the Master Constraint

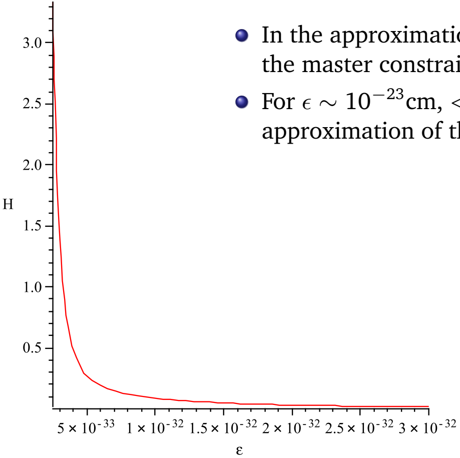
Study of the $\langle \hat{\mathbb{H}} \rangle_{\text{min}}$ for different choices of $\epsilon\,(\ell_p)$ and different powers of ϵ/ℓ_{P} :



- In the approximation studied: no continuum limit, but the master constraint quickly drops to zero for $\epsilon > \ell_p$.

Minimizing the Master Constraint

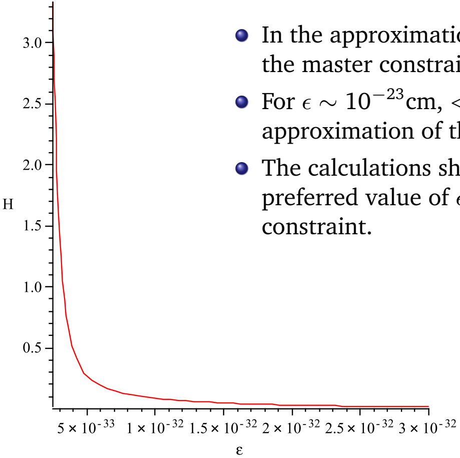
Study of the $\langle\hat{\mathbb{H}}\rangle_{\text{min}}$ for different choices of $\epsilon\,(\ell_p)$ and different powers of ϵ/ℓ_{P} :



- In the approximation studied: no continuum limit, but the master constraint quickly drops to zero for $\epsilon > \ell_p$.
- For $\epsilon \sim 10^{-23}$cm, $< \hat{\mathbb{H}} >\sim 10^{-20}$. Very good approximation of the the continuum theory.

Minimizing the Master Constraint

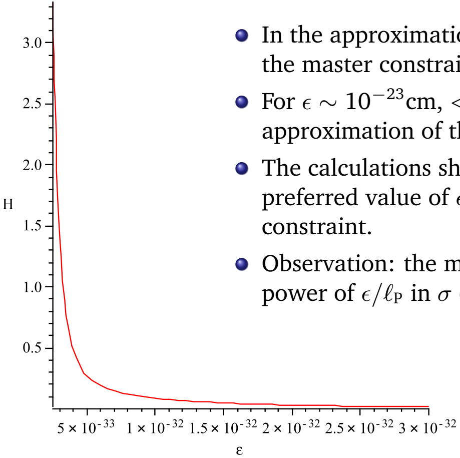
Study of the $\langle\hat{\mathbb{H}}\rangle_{\text{min}}$ for different choices of ϵ (ℓ_p) and different powers of ϵ/ℓ_{P} :



- In the approximation studied: no continuum limit, but the master constraint quickly drops to zero for $\epsilon > \ell_p$.
- For $\epsilon \sim 10^{-23}$cm, $< \hat{\mathbb{H}} >\sim 10^{-20}$. Very good approximation of the the continuum theory.
- The calculations show that there exists a genuine preferred value of ϵ that minimizes the master constraint.

Minimizing the Master Constraint

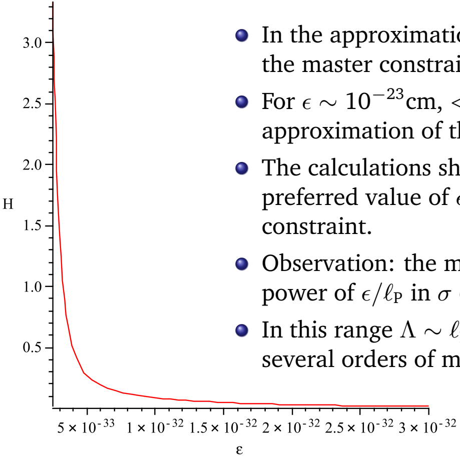
Study of the $\langle\hat{\mathbb{H}}\rangle_{\mathrm{min}}$ for different choices of $\epsilon\,(\ell_p)$ and different powers of $\epsilon/\ell_{\mathrm{P}}$:



- In the approximation studied: no continuum limit, but the master constraint quickly drops to zero for $\epsilon > \ell_p$.
- For $\epsilon \sim 10^{-23}$cm, $< \hat{\mathbb{H}} >\sim 10^{-20}$. Very good approximation of the the continuum theory.
- The calculations show that there exists a genuine preferred value of ϵ that minimizes the master constraint.
- Observation: the minimum $< \hat{\mathbb{H}} >$ is achieved for a power of $\epsilon/\ell_{\mathrm{P}}$ in σ close to 2 and $\epsilon \sim 10^{13}\ell_{\mathrm{P}}$.

Minimizing the Master Constraint

Study of the $\langle\hat{\mathbb{H}}\rangle_{\min}$ for different choices of $\epsilon\,(\ell_p)$ and different powers of ϵ/ℓ_P :



- In the approximation studied: no continuum limit, but the master constraint quickly drops to zero for $\epsilon > \ell_p$.
- For $\epsilon \sim 10^{-23}$cm, $< \hat{\mathbb{H}} > \sim 10^{-20}$. Very good approximation of the the continuum theory.
- The calculations show that there exists a genuine preferred value of ϵ that minimizes the master constraint.
- Observation: the minimum $< \hat{\mathbb{H}} >$ is achieved for a power of ϵ/ℓ_P in σ close to 2 and $\epsilon \sim 10^{13}\ell_P$.
- In this range $\Lambda \sim \ell_P^2/\epsilon^2$ would not be of Planck scale but several orders of magnitude smaller.

Outline

1 The 3+1 Spherically Symmetric Model
- The Hamiltonian
- Quantization and the Master Constraint
- States
- The Propagator

Is Fock Vacuum Appropriate?

- Is it appropriate to use this vacuum in the state $|\psi_{\vec{\sigma}}^{\text{trial}}\rangle$ to compute the propagator?

Is Fock Vacuum Appropriate?

- Is it appropriate to use this vacuum in the state $|\psi_{\vec{\sigma}}^{\text{trial}}\rangle$ to compute the propagator?
- To answer: show that the corrections to the Fock representation due to holonomization are small enough for our purpose.

Is Fock Vacuum Appropriate?

- Fully holonomized Hamiltonian

$$H_{\text{matt}}(i) = \frac{H^{(1)}(i)}{\left(E^{\varphi}(i)\right)^2} + \frac{H^{(2)}(i)\sin\left(\rho K_{\varphi}(i)\right)}{\rho E^{\varphi}(i)},$$

where,

$$\begin{aligned} H^{(1)}(i) =& \frac{\epsilon}{2} P_f^2(i) x(i)^2 + \frac{\epsilon^3 \sin^2\left(\beta f(i)\right)}{2\beta^2} - \frac{\epsilon^2 x(i)}{\beta^2} \sin\left[\beta f(i) \sin\left(\beta\left(f(i+1) - f(i)\right)\right)\right] \\ &+ \frac{\epsilon x(i)^2}{2\beta^2} \sin^2\left(\beta\left(f(i+1) - f(i)\right)\right), \\ H^{(2)}(i) =& \frac{P_f(i)}{\beta}\left(\epsilon \sin\left(\beta f(i)\right) - x(i)\sin\left(\beta\left(f(i+1) - f(i)\right)\right)\right) \end{aligned}$$

with β being the holonomization parameter.

Is Fock Vacuum Appropriate?

- Expanding in β and keeping the two lowest orders:

$$H^{(1)}(i) = H^{(1)}_{\text{lead}}(i) + H^{(1)}_{\text{corr}}(i),$$
$$H^{(2)}(i) = H^{(2)}_{\text{lead}}(i) + H^{(2)}_{\text{corr}}(i)$$

leading orders correspond to the Fock terms; correction terms of order β^2 are holonomization corrections.

Is Fock Vacuum Appropriate?

- Expanding in β and keeping the two lowest orders:

$$H^{(1)}(i) = H^{(1)}_{\text{lead}}(i) + H^{(1)}_{\text{corr}}(i),$$
$$H^{(2)}(i) = H^{(2)}_{\text{lead}}(i) + H^{(2)}_{\text{corr}}(i)$$

leading orders correspond to the Fock terms; correction terms of order β^2 are holonomization corrections.

- The correction part of the master constraint:

$$\begin{aligned}\mathbb{H}_{\text{corr}}(i) =& c_1(i)H^{(1)}_{\text{corr}}(i) + c_2(i)H^{(2)}_{\text{corr}}(i) + c_{00}(i) + c_{11}(i)\left(H^{(1)}_{\text{lead}}(i)H^{(1)}_{\text{corr}}(i)\right) \\ &+ c_{12}(i)\left(H^{(1)}_{\text{lead}}(i)H^{(2)}_{\text{corr}}(i) + H^{(2)}_{\text{lead}}(i)H^{(1)}_{\text{corr}}(i)\right) + c_{22}(i)\left(H^{(2)}_{\text{lead}}(i)H^{(2)}_{\text{corr}}(i)\right)\end{aligned}$$

Is Fock Vacuum Appropriate?

- Expanding in β and keeping the two lowest orders:

$$H^{(1)}(i) = H^{(1)}_{\text{lead}}(i) + H^{(1)}_{\text{corr}}(i),$$
$$H^{(2)}(i) = H^{(2)}_{\text{lead}}(i) + H^{(2)}_{\text{corr}}(i)$$

  leading orders correspond to the Fock terms; correction terms of order β^2 are holonomization corrections.

- The correction part of the master constraint:

$$\begin{aligned}\mathbb{H}_{\text{corr}}(i) =& c_1(i)H^{(1)}_{\text{corr}}(i) + c_2(i)H^{(2)}_{\text{corr}}(i) + c_{00}(i) + c_{11}(i)\left(H^{(1)}_{\text{lead}}(i)H^{(1)}_{\text{corr}}(i)\right) \\ &+ c_{12}(i)\left(H^{(1)}_{\text{lead}}(i)H^{(2)}_{\text{corr}}(i) + H^{(2)}_{\text{lead}}(i)H^{(1)}_{\text{corr}}(i)\right) + c_{22}(i)\left(H^{(2)}_{\text{lead}}(i)H^{(2)}_{\text{corr}}(i)\right)\end{aligned}$$

- Show that $\int dx \langle \psi^{\text{trial}}_{\vec{\sigma}} | \mathbb{H}_{\text{corr}} | \psi^{\text{trial}}_{\vec{\sigma}} \rangle$ is very small.

Is Fock Vacuum Appropriate?

- The correction terms give

$$\mathbf{H}_{\mathrm{corr}} = \int_{\epsilon}^{L} \langle \mathbb{H}_{\mathrm{corr}} \rangle dx \sim \frac{\ell_p^5}{\epsilon^4} \beta^2$$

Is Fock Vacuum Appropriate?

- The correction terms give

$$\mathbf{H}_{\mathrm{corr}} = \int_{\epsilon}^{L} \langle \mathbb{H}_{\mathrm{corr}} \rangle dx \sim \frac{\ell_p^5}{\epsilon^4} \beta^2$$

- The leading order we had before (Fock case) was

$$\mathbf{H}_{\mathrm{lead}} = \int_{\epsilon}^{L} \langle \mathbb{H}_{\mathrm{lead}} \rangle dx \sim \frac{\ell_p^3}{\epsilon^2}$$

Is Fock Vacuum Appropriate?

- The correction terms give

$$\mathbf{H}_{\text{corr}} = \int_{\epsilon}^{L} \langle \mathbb{H}_{\text{corr}} \rangle dx \sim \frac{\ell_p^5}{\epsilon^4} \beta^2$$

- The leading order we had before (Fock case) was

$$\mathbf{H}_{\text{lead}} = \int_{\epsilon}^{L} \langle \mathbb{H}_{\text{lead}} \rangle dx \sim \frac{\ell_p^3}{\epsilon^2}$$

- Thus for $\epsilon \gg \ell_p \Rightarrow \mathbf{H}_{\text{corr}} \ll \mathbf{H}_{\text{lead}}$: the vacuum of the theory is well approximated by the tensor product $|\psi_{\vec{\sigma}}^{\text{trial}}\rangle = |\psi_{\vec{\sigma}}\rangle \otimes |0\rangle$: we can continue to use it to compute the propagator.

The Choices of Polymerization

- The discretized Hamiltonian written in a more symmetric way

$$H = \sum_i \frac{P_f(i)^2}{2\epsilon} - \frac{(f(i+1) + f(i-1) - 2f(i))f(i)}{2\epsilon}$$

The Choices of Polymerization

- The discretized Hamiltonian written in a more symmetric way

$$H = \sum_i \frac{P_f(i)^2}{2\epsilon} - \frac{(f(i+1) + f(i-1) - 2f(i))f(i)}{2\epsilon}$$

- Several choices for polymerization.

The Choices of Polymerization

- The discretized Hamiltonian written in a more symmetric way

$$H = \sum_i \frac{P_f(i)^2}{2\epsilon} - \frac{(f(i+1) + f(i-1) - 2f(i))f(i)}{2\epsilon}$$

- Several choices for polymerization.
- We go for two choices: the field f itself; the momentum P_f. They do not lead to equivalent theories. Also polymerizing the momentum yields a theory that in the continuum limit is not polymeric.

Polymerizing the Scalar Field

- The polymerized Hamiltonian in this case

$$H = \sum_i \left(\frac{P_f(i)^2}{2\epsilon} - \frac{\sin\left(\beta\left(f(i+1) + f(i-1) - 2f(i)\right)\right)\sin(\beta f(i))}{2\epsilon\beta^2} \right)$$

Polymerizing the Scalar Field

- The polymerized Hamiltonian in this case

$$H = \sum_i \left(\frac{P_f(i)^2}{2\epsilon} - \frac{\sin\left(\beta\left(f(i+1) + f(i-1) - 2f(i)\right)\right)\sin(\beta f(i))}{2\epsilon\beta^2} \right)$$

- Perturbative expansion in β yields $H = H_0 + H_{\text{int}}$ with

$$H_0 = \sum_i \left(\frac{P_f(i)^2}{2\epsilon} - \frac{f(i)\left(f(i+1) + f(i-1) - 2f(i)\right)}{2\epsilon} \right)$$

$$H_{\text{int}} = \sum_i \frac{1}{2\epsilon} \left[\frac{1}{6} f(i)\left(f(i+1) + f(i-1) - 2f(i)\right)^3 + \frac{1}{6} f(i)^3 \left(f(i+1) + f(i-1) - 2f(i)\right) \right] \beta^2$$

Polymerizing the Scalar Field

- The propagator to leading order is

$$G^{(2)}(j,t,k,t') = G^{(0)}(j,t,k,t') + \frac{i^2}{2!}\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \times \sum_{j'=-N}^{N}\sum_{k'=-N}^{N} \langle 0|T\left(f(j,t)f(k,t')H_{\text{int}}(j',t_1)H_{\text{int}}(k',t_2)\right)|0\rangle$$

where we use $a,\dots,j,k$ for direct space and $m,n,\dots,z$ for momentum space.

Polymerizing the Scalar Field

- Going from direct to momentum representation $(\ldots, j, k \to m, n, \ldots)$ and from time t to ω space, with $p(n) \equiv \pi n/L$ and $L = N\epsilon$ yields

$$
\begin{aligned}
G^{(2)}(n_1,\omega_1,n_2,\omega_2) =& G^{(0)}(n_1,\omega_1,n_2,\omega_2) \\
&+ \left[\frac{\alpha_1\beta^4}{\epsilon^2} + \beta^4\alpha_2 p(n_1)^2\right] \frac{4\pi i}{\epsilon} \frac{\delta(\omega_1-\omega_2)\left(\delta_{n_1,n_2}-\delta_{n_1,-n_2}\right)}{\left(\omega_1^2 - p(n_1)^2 + i\sigma\right)^2} \\
\approx& \frac{4\pi i}{\epsilon} \frac{1}{\omega_1^2 - p(n_1)^2\left(1+\alpha_2\beta^4\right) - \frac{\alpha_1\beta^4}{\epsilon^2} + i\sigma} \\
&\times \left(\delta_{n_1,n_2}-\delta_{n_1,-n_2}\right)\delta(\omega_1-\omega_2)
\end{aligned}
$$

to lowest order in $p(n)$. With α_1 and α_2 constants of order one.

Polymerizing the Momentum Field

- The same method of calculations for the polymerizing the momentum in the Hamiltonian

$$H = \sum_i \frac{\sin^2\left(\beta P_f(i)\right)}{2\beta^2\epsilon} - \frac{\left(f(i+1)+f(i-1)-2f(i)\right)f(i)}{2\epsilon}$$

yields

$$\begin{aligned} G^{(2)}(n_1,\omega_1,n_2,\omega_2) =& G^{(0)}(n_1,\omega_1,n_2,\omega_2) \\ & + \beta^4\alpha_2 p(n_1)^2 \frac{4\pi i}{\epsilon} \frac{\delta(\omega_1-\omega_2)\left(\delta_{n_1,n_2}-\delta_{n_1,-n_2}\right)}{\left(\omega_1^2-p(n_1)^2+i\sigma\right)^2} \\ \approx & \frac{4\pi i}{\epsilon} \frac{1}{\omega_1^2-p(n_1)^2\left(1+\alpha_2\beta^4\right)+i\sigma} \\ & \times \left(\delta_{n_1,n_2}-\delta_{n_1,-n_2}\right)\delta(\omega_1-\omega_2) \end{aligned}$$

to lowest order in $p(n)$. With α_2 constant of order one.

Lorentz Invariance Violation

- In both of the propagators we derived, generically there will be higher powers of p in the denominator (we only kept the lowest). They are in the class of Hořava's "Gravity at the Lifshitz point":

$$\frac{1}{\omega^2 - c^2\mathbf{k}^2 - G(\mathbf{k}^2)^z}$$

which signal Lorentz invariance violation.

Lorentz Invariance Violation

- Two distinct origins for the modifications of the dispersion relation:

Lorentz Invariance Violation

- Two distinct origins for the modifications of the dispersion relation:
  - One stems from the **discreteness**: remnant from the discretization procedure. Because the state that minimizes the expectation value of the master constraint does so for a finite lattice spacing.

Lorentz Invariance Violation

- Two distinct origins for the modifications of the dispersion relation:
  - One stems from the **discreteness**: remnant from the discretization procedure. Because the state that minimizes the expectation value of the master constraint does so for a finite lattice spacing.
  - The other comes from the **polymerization** (the factor β).

Lorentz Invariance Violation

- Two distinct origins for the modifications of the dispersion relation:
    - One stems from the **discreteness**: remnant from the discretization procedure. Because the state that minimizes the expectation value of the master constraint does so for a finite lattice spacing.
    - The other comes from the **polymerization** (the factor β).

Corrections due to Polymerization

The order in β at which corrections appear can be shifted (and made small) arbitrarily by choosing suitable polymerizations of the theory: e.g. the corrections to the dispersion relation can be made to be of order β^8 instead of β^4, etc.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.
- The resulting propagators are within Horava's class of Lorentz violating theories.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.
- The resulting propagators are within Horava's class of Lorentz violating theories.
- The scalar field may acquire a mass.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.
- The resulting propagators are within Horava's class of Lorentz violating theories.
- The scalar field may acquire a mass.
- The value of the parameter β is unclear.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.
- The resulting propagators are within Horava's class of Lorentz violating theories.
- The scalar field may acquire a mass.
- The value of the parameter β is unclear.
- One may feel tempted to deduce some strong deductions, e.g. make this responsible for the Higgs mass, but we should recall that we are in a very limited context, spherical symmetry.

Summary

- The study of spherically symmetric gravity coupled to a scalar field using techniques of loop quantum gravity has progressed to the point where we can compute propagators.
- The resulting propagators are within Horava's class of Lorentz violating theories.
- The scalar field may acquire a mass.
- The value of the parameter β is unclear.
- One may feel tempted to deduce some strong deductions, e.g. make this responsible for the Higgs mass, but we should recall that we are in a very limited context, spherical symmetry.
- The form of the Lorentz violation depends on how one polymerizes.

References

This work is done under supervision of Rodolfo Gambini and in collaboration with Jorge Pullin (LSU). For more details see:

- R. Gambini, J. Pullin, S. Rastgoo, Classical Quant. Grav. **28**, 155005 (2011) [arXiv:1106.1417].
- R. Gambini, J. Pullin, S. Rastgoo, Gen. Rel. Grav. **43** 3569-3592 (2011) [arXiv:1105.0667].
- R. Gambini, J. Pullin, S. Rastgoo, Classical Quant. Grav. **27**, 025002 (2010) [arXiv:0909.0459].
- R. Gambini, J. Pullin, S. Rastgoo, Classical Quant. Grav. **26**, 215011 (2009) [arXiv:0906.1774].