Qualitative Effective Dynamics in Bianchi IX Loop Quantum Cosmology

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Motivation

This effective theory comes from the construction of the full quantum theory and we expect that it gives some insights about the quantum dynamics of semiclassical states. The accuracy of the effective equations has been established in the isotropic cases and thus we expect that they should give an excellent approximation of the full quantum evolution for semiclassical states.

- We want to study homogeneous and anisotropic universes, these kind of universes are classified as Bianchi models. We are interested in the Bianchi IX model which the spacetime is like $M = \Sigma \times \mathbb{R}$ where Σ is a spatial 3-manifold which can be identified by the symmetry group $SU(2)$. These models are of interest to the issue of singularity resolution (BKL conjecture).
- In particular, we analyzed the numerical solutions of the effective equations that come from the improved LQC dynamics of the Bianchi IX model. We choose a massless scalar field as the matter source. When the momentum of the field is zero we reduce to the vaccum case.

- **Bianchi I** have a 3-dimensional group of symmetries generated by three translations. Its spatial 3-manifold can be identified with $\mathbb{R}^3$. The metric is

$$ds^2 = -N^2 \mathrm{d}t^2 + a_1(t)^2 \mathrm{d}^2 x + a_2(t)^2 \, \mathrm{d}y^2 + a_3(t)^2 \, \mathrm{d}z^2 \tag{1}$$

- **Bianchi II** have a 3-dimensional group of symmetries generated by two translations and a rotation on a null 2-plane. The metric is

$$ds^2 = -N^2 \mathrm{d}t^2 + a_1(t)^2 \, (\mathrm{d}x - \alpha z \, \mathrm{d}y)^2 + a_2(t)^2 \, \mathrm{d}y^2 + a_3(t)^2 \, \mathrm{d}z^2 \tag{2}$$

  where α is a switch (Bianchi I $\alpha = 0$, Bianchi II $\alpha = 1$).
- **Bianchi IX** metric can be construct from the fiducial co-triads

$$\begin{aligned} {}^o\omega_a^1 &= \sin\beta \sin\gamma (\mathrm{d}\alpha)_a + \cos\gamma (\mathrm{d}\beta)_a, \\ {}^o\omega_a^2 &= -\sin\beta \cos\gamma (\mathrm{d}\alpha)_a + \sin\gamma (\mathrm{d}\beta)_a, \\ {}^o\omega_a^3 &= \cos\beta (\mathrm{d}\alpha)_a + (\mathrm{d}\gamma). \end{aligned} \tag{3}$$

  with physical co-triads $\omega_a^i = a^i(t)^o\omega_a^i$ and 3-metric $q_{ab} := \omega_a^i \omega_{bi}$. Then the spacetime metric is

$$g_{\mu\nu} = -n_\mu n_\nu + q_{\mu\nu} \tag{4}$$

Observables

- Directional scale factors, $a_i = L_i^{-1}\sqrt{\frac{p_j p_k}{p_i}}$.
- Hubble parameters, $H_i = \frac{a_i'}{a_i} = \frac{1}{2}\left(\frac{p_j'}{p_j} + \frac{p_k'}{p_k} - \frac{p_i'}{p_i}\right)$.
- Expansion, $\theta = \frac{V'}{V} = H_1 + H_2 + H_3$.
- Matter density, $\rho = \frac{p_\phi^2}{2V^2} = \frac{p_\phi^2}{2p_1 p_2 p_3}$.
- Density parameter, $\Omega = \frac{24\pi G\rho}{\theta^2}$.
- Shear, $\sigma^2 = \frac{1}{3}[(H_1 - H_2)^2 + (H_1 - H_3)^2 + (H_2 - H_3)^2]$.
- Shear parameter, $\Sigma^2 = \frac{3\sigma^2}{2\theta^2}$, where $\Omega + \Sigma^2 = 1$ for the classical Bianchi I.
- Ricci scalar R, Kretschmann scalar $K = R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu}$ and the square of the Weyl curvature $C_{\alpha\beta\mu\nu}C^{\alpha\beta\mu\nu}$.
- Kasner exponents, $k_i = \frac{H_i}{|\theta|}$. For Bianchi I, $k_1 + k_2 + k_3 = \pm 1$ and $k_1^2 + k_2^2 + k_3^2 + k_\phi^2 = 1$

Effective Hamiltonian (Edward)

$$
\begin{aligned}
\mathcal{H} = & -\frac{p_1 p_2 p_3}{8\pi G\gamma^2\lambda^2}\left(\sin\bar{\mu}_1 c_1 \sin\bar{\mu}_2 c_2 + \sin\bar{\mu}_2 c_2 \sin\bar{\mu}_3 c_3 + \sin\bar{\mu}_3 c_3 \sin\bar{\mu}_1 c_1\right) \\
& -\frac{\vartheta}{8\pi G\gamma^2\lambda}\left(\frac{(p_1 p_2)^{3/2}}{\sqrt{p_3}}\sin\bar{\mu}_3 c_3 + \frac{(p_2 p_3)^{3/2}}{\sqrt{p_1}}\sin\bar{\mu}_1 c_1 + \frac{(p_3 p_1)^{3/2}}{\sqrt{p_2}}\sin\bar{\mu}_2 c_2\right) \\
& -\frac{\vartheta^2(1+\gamma^2)}{32\pi G\gamma^2}\left[2(p_1^2+p_2^2+p_3^2) - \left(\frac{p_1 p_2}{p_3}\right)^2 - \left(\frac{p_2 p_3}{p_1}\right)^2 - \left(\frac{p_3 p_1}{p_2}\right)^2\right] \\
& +\frac{p_\phi^2}{2} \approx 0
\end{aligned}
$$

where $\vartheta = (2\pi^2)^{1/3}$, $\lambda^2 = 4\sqrt{3}\pi\gamma l_{\rm Pl}^2$ and

$$
\bar{\mu}_i = \lambda\sqrt{\frac{|p_i|}{|p_j p_k|}}, \quad \text{with} \quad i \neq j \neq k.
$$

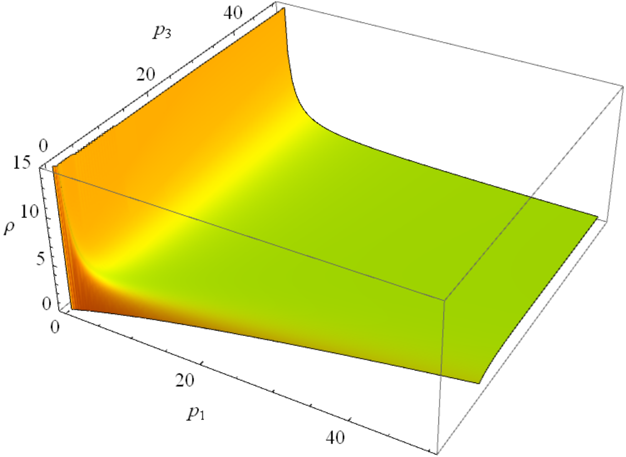
Effective Hamiltonian (Asieh)

$$
\begin{aligned}
\mathcal{H} &= -\frac{V^4 A(V) h^6(V)}{8\pi G V_c^6 \gamma^2 \lambda^2} \left(\sin \bar{\mu}_1 c_1 \sin \bar{\mu}_2 c_2 + \sin \bar{\mu}_1 c_1 \sin \bar{\mu}_3 c_3 + \sin \bar{\mu}_2 c_2 \sin \bar{\mu}_3 c_3 \right) \\
&\quad - \frac{\vartheta A(V) h^4(V)}{4\pi G V_c^4 \gamma^2 \lambda} \left(p_1^2 p_2^2 \sin \bar{\mu}_3 c_3 + p_2^2 p_3^2 \sin \bar{\mu}_1 c_1 + p_1^2 p_3^2 \sin \bar{\mu}_2 c_2 \right) \\
&\quad - \frac{\vartheta^2 (1+\gamma^2) A(V) h^4(V)}{8\pi G V_c^4 \gamma^2} \times \\
&\qquad \left(2V[p_1^2 + p_2^2 + p_3^2] - \left[(p_1 p_2)^4 + (p_1 p_3)^4 + (p_2 p_3)^4 \right] \frac{h^6(V)}{V_c^6} \right) \\
&\quad + \frac{h^6(V) V^2}{2 V_c^6} p_\phi^2 \approx 0
\end{aligned}
$$

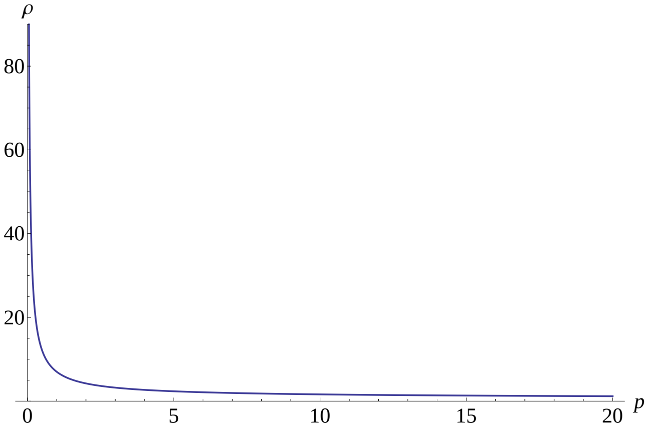
with $V_c = 2\pi\gamma\lambda\ell_p^2$ and

$$
\begin{aligned}
h(V) &= \sqrt{V + V_c} - \sqrt{|V - V_c|} && (5) \\
A(V) &= \frac{1}{2V_c} (V + V_c - |V - V_c|) && (6)
\end{aligned}
$$

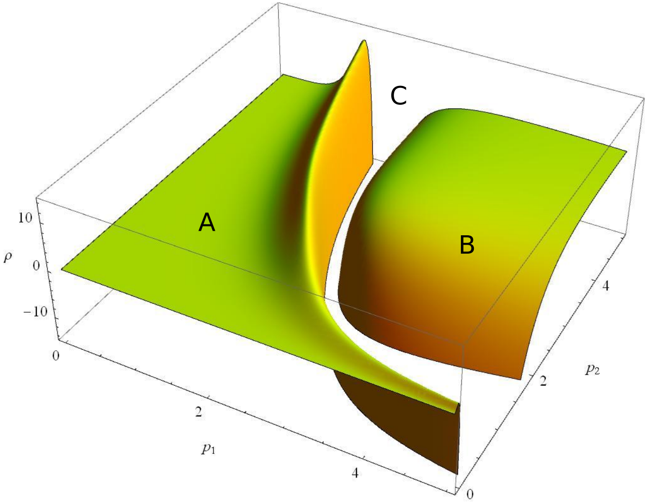
Maximal Density (Edward)



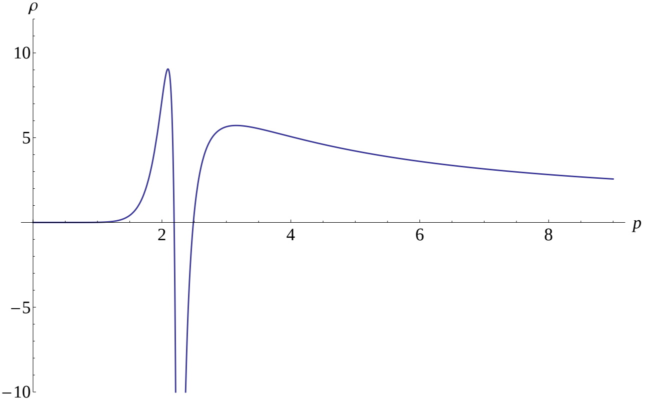
Maximal Density (Edward)



Maximal Density (Asieh)



Maximal Density (Asieh)



Classical Limit

The universe can be seen as a particle moving in a potential that presents reflections at exponential walls. The classical potential in the classical constraint, in terms of Misner variables is

$$W = \frac{1}{2}e^{-4\Omega}\left(e^{-4\beta_+} - 4e^{-\beta_+}\cosh\sqrt{3}\beta_- + 2e^{-2\beta_+}[\cosh 2\sqrt{3}\beta_- - 1]\right), \qquad (7)$$

where $\Omega = -\frac{1}{3}\log V$ and the anisotropies $\beta_\pm$ are defined via

$$a_1 = e^{-\Omega+(\beta_+ + \sqrt{3}\beta_-)/2}, \quad a_2 = e^{-\Omega+(\beta_+ - \sqrt{3}\beta_-)/2}, \quad a_3 = e^{-\Omega-\beta_+} \qquad (8)$$

The modified potential as a function of p_i is

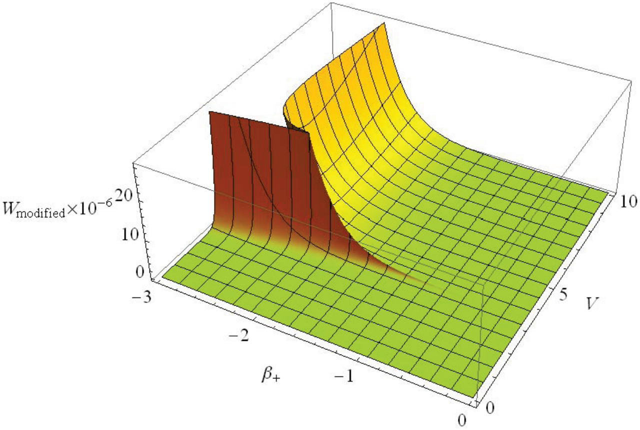
$$W_{\text{eff}} = -\frac{V^2 A(V) h^4(V)}{V_c^4}\left(p_1^2 + p_2^2 + p_3^2 - \left[(p_1 p_2)^4 + (p_1 p_3)^4 + (p_2 p_3)^4\right]\frac{h^6(V)}{2VV_c^6}\right)$$

For a simple case, when $\beta_- = 0$ and $\beta_+ \to -\infty$, the potentials are

$$W \sim \frac{1}{2}e^{-4\Omega-4\beta_+} \qquad \text{and} \qquad W_{\text{eff}} \sim \frac{1}{2V_c^9}e^{-52\Omega-4\beta_+} \qquad (9)$$

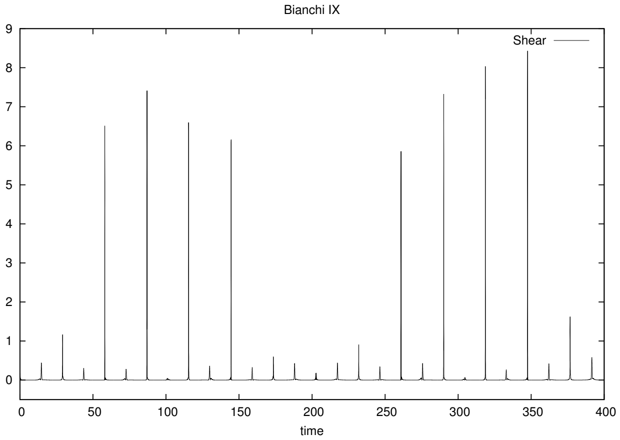
The β_+-dependency of both potentials is the same. Thus, we have an exponential wall for the modified potential, too.

Effective Potencial

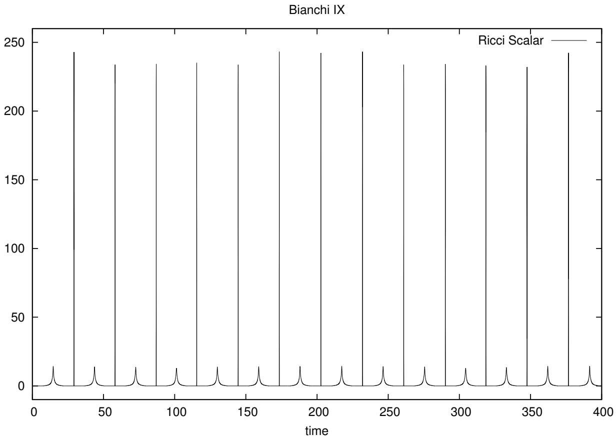


Modified potential with $\beta_- = 0$ ($a_1 = a_2$).

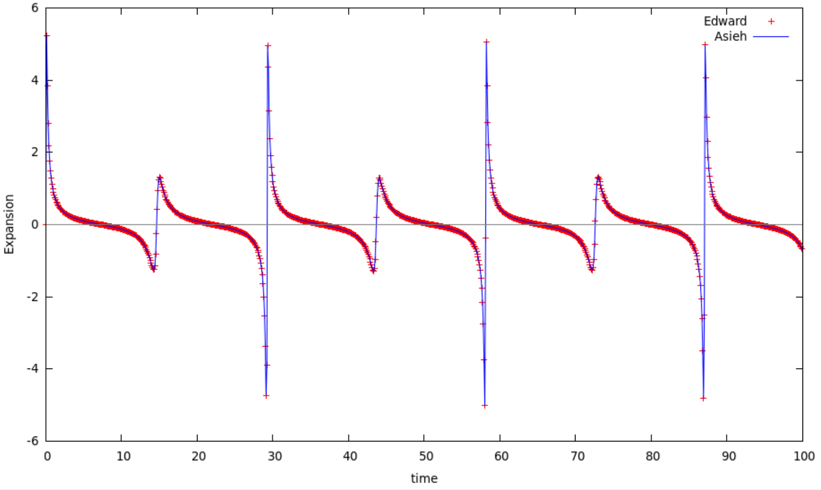
Shear Evolution (Asieh)



Ricci Scalar Evolution (Asieh)



Comparison for the two Effective Theories



Preliminary Results

- All solutions have a bounce. In other words, singularities are resolved. In Bianchi IX, there is an infinite number of bounces and recollapses.
- If the inverse triad corrections are not included, then the geometric scalars (θ, σ^2, ρ) are not absolutely bounded. But if the inverse triad corrections are included then, on each solution, the geometric scalars are bounded but there is not an absolute bound for all the solutions.
- Bianchi I, II and therefore the isotropic case k=0 are limiting cases of Bianchi IX, but they are not contained within Bianchi IX. While the isotropic FRW k=1 is contained within Bianchi IX only if the inverse triad corrections are not included.
- The Kasner exponents tell us about the Bianchi I transitions (if they exist) and particularly in Bianchi IX, they are used to study the BKL behavior in the vacuum case.
- The potential wall does not disappear and we have, potentially, chaotic behavior near the classical singularity. However, if we start from large volumes there will be a lower bound for volume. Since there are no large anisotropies near the smallest allowed volume, the solutions will *not* exhibit chaotic behavior.

Thanks

Questions

- Is solve the chaotic behavior when there is not the inverse triad corrections?
- Does the effective dynamics of the anisotropic models reproduce the full quantum dynamics for semiclassical states?
- Loop quantum cosmology has provided a complete description of the quantum dynamics in the case of isotropic cosmological models and singularity resolution has been shown to be generic. A pressing question is, can these results be generalized to anisotropic models?
- With the study of these anisotropic models, a question that still arises is whether this bouncing non-singular behavior is generic for inhomogeneous configurations. That is, are we a step forward toward generic quantum singularity resolution?