BLACK HOLES FROM

LOOP QUANTUM GRAVITY'S PERSPECTIVE

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Quantum gravity attempts to bring together general relativity and quantum theory in a single framework. The two pillars of XX century physics are incompatible!

What clues can we find to guide us in that search?

Black holes provide one of such keys.
Even when they are prominent classical objects, they might provide the clue to quantum gravity.

How?

Black Holes are believed to possess thermodynamical properties, such as temperature and entropy, that are associated to macroscopic properties.

But the origin of this behavior involves quantum physics ($\hbar$ is present).

Do we need to solve quantum gravity to explain this?

Or, can we use this clue as a guide in our search?

PLAN OF THE COURSE

1. Why quantum gravity?
2. Loop Quantum Gravity
3. Isolated Horizons
4. Quantum Horizon Geometry
5. Black Holes and Entropy
6. Loop Quantum Black Holes
7. Entropy
8. Recent Results and Outlook

WHY QUANTUM GRAVITY ?

We have a strange situation in fundamental physics:

General Relativity is an excellent theory to describe all macroscopic phenomena. Even the Universe.

But all microscopic phenomena are described by a new set of rules.

General Relativity does not follow the Quantum rules.

QFT methods for dealing with other interactions do *not* extend to general relativity.

But we need to have a unified description of physics, not two theories that are valid in different regimes!

PLANCK SCALE

Already Planck noticed that a combination of the three fundamental constants

$$c \qquad ; \qquad G \qquad \text{and} \qquad \hbar$$

yields a new scale:

$$\ell_{\rm Pl} = \sqrt{\frac{\hbar\, G}{c^3}} = 1.616252 \times 10^{-35} \mathrm{m}$$

Also,

$$m_{\rm Pl} = \sqrt{\frac{\hbar\, c}{G}} = 2.17644 \times 10^{-8} \mathrm{kg} \quad ; \quad t_{\rm Pl} = \sqrt{\frac{\hbar\, G}{c^5}} = 5.39124 \times 10^{-44} \mathrm{s}$$

One expects that, at this scale, quantum gravity effects dominate.

When or where do we expect to approach this regime?

In ordinary phenomena we are always very far from that scale. Safe to ignore quantum gravitational effects.

Only when gravitational interaction dominates, and causes total collapse (of a star), one approches densities and temperatures in the Planck scale.

And near the Big Bang.

Is there a another macroscopic situation where there could be gravitational quantum effects?

BLACK HOLES AND QUANTUM GRAVITY ?

Black Holes are, as Chandrasekhar used to say:

"... the most perfect objects there are in The Universe: the only elements in their construction are our concepts of space and time. Since GR predicts a single family of solutions, they are the simplest as well." **They are the crown of classical physics in terms of their simplicity and beauty.**

But, Bekenstein and Hawking told us that :

i) Black Holes satisfy some 'thermodynamic-like laws'.

$$\delta M = \frac{\kappa}{8\pi G}\,\delta A \;\;\Rightarrow\;\; M \leftrightarrow E, \quad \kappa \leftrightarrow T\,, \quad A \leftrightarrow S$$

ii) When one invokes quantum mechanics ($\hbar$) then something weird happens:

$$E = M \qquad ; \qquad T = \frac{\kappa\,\hbar}{2\pi}\ ,$$

and

$$S = \frac{A}{4\,G\hbar}$$

Black holes seem to have thermodynamic properties!

What are then the underlying degrees of freedom responsible for entropy?

The standard wisdom is that only with a full marriage of the Quantum and Gravity will we be able to understand this.

Different approaches:

- **String Theory**
- **Causal Sets**
- **Entanglement Entropy**
- **Loop Quantum Gravity: This Meeting!!**

QUESTIONS TO BE ADDRESSED

- **We need a quantum description for black holes**
- **How do we characterize black holes in equilibrium?**
- **Can we define quantum horizon states?**
- **Which states should we count?**
- **How does the entropy behave?**
- **Large BH: Bekenstein-Hawking entropy**

Horizons in Equilibrium.

Physically, one is interested in describing black holes in equilibrium. That is, equilibrium of the horizon, not the exterior. Can one capture that notion via boundary conditions?

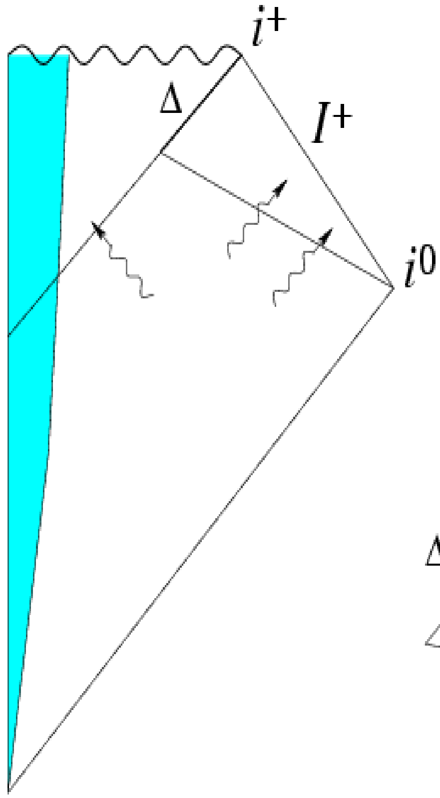
Yes! Answer: Isolated Horizons

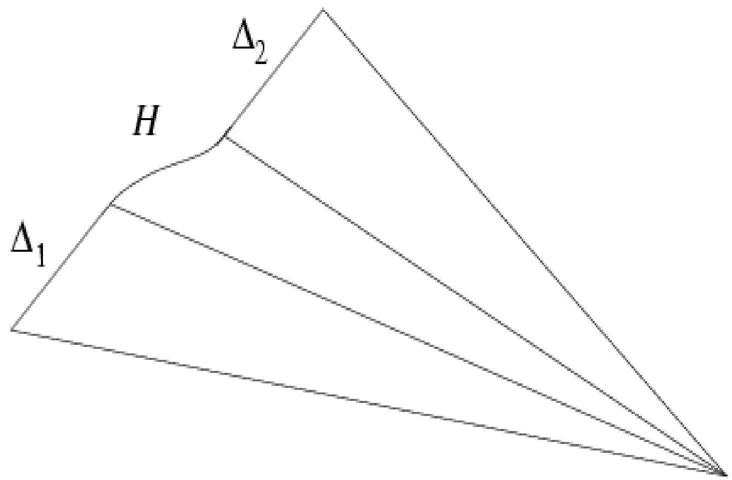
Isolated horizon boundary conditions are imposed on an inner boundary of the region under consideration.

(Ashtekar, AC, Krasnov, ATMP 3, 419, 2000)

The interior of the horizon is cut out.
In this a physical boundary?
No! but one can ask whether one can make sense of it:

What is then the physical interpretation of the boundary?





• The null boundary Δ, the 3-D isolated horizon, provides an effective description of the degrees of freedom of the inside region, that is cut out in the formalism.

ISOLATED HORIZONS

An isolated horizon is a null, non-expanding horizon Δ with some notion of translational symmetry along its generators. Technically we consider Weakly Isolated Horizons (WIH). We will also restrict ourselves to Type I, spherically symmetric, horizons (see Engle's talk for type II). There are two main consequences of the boundary conditions:

- **The gravitational degrees of freedom induced on the horizon are captured in a $U(1)$ connection,**

$$W_a = -\frac{1}{2}\,\Gamma^i_a\, r_i$$

• The total symplectic structure of the theory (and this is true even when most matter is present) gets split as,

$$\Omega_{\rm tot} = \Omega_{\rm bulk} + \Omega_{\rm hor}$$

with

$$\Omega_{\rm hor} = \frac{a_0}{8\pi^2 G\gamma} \oint_S {\rm d}W \wedge {\rm d}W'$$

• The 'connection part' and the 'triad part' at the horizon must satisfy the condition,

$$F_{ab} = -\frac{2\pi\,\gamma}{a_0}\, E^i_{ab}\, r_i,$$

which is called the 'horizon constraint'.

CONSTRAINTS

The Hamiltonian formalism tells us is a natural way what is gauge and what not. In particular, with regard to the constraints we know that:

• The relation between curvature and triad, the horizon constraint, is equivalent to Gauss' law.

• Diffeomorphims that leave S invariant are gauge (their vector field are tangent to S).

• The scalar constraint must have $N|_{\mathrm{hor}} = 0$. Thus, the scalar constraint leaves the horizon untouched; any gauge and diff-invariant observable *is* a Dirac observable!

In the quantum theory of the horizon we have to implement these facts.

THE BULK: LOOP QUANTUM GRAVITY

A canonical description in terms of $SU(2)$ Yang-Mills:

$$A_a^i \quad SU(2) \text{ connection} \quad ; \quad E_i^a \quad \text{triad}$$

with $A_a^i = \Gamma_a^i + \gamma\, K_a^i$. Loop Quantum gravity on a 3-dimensional space without boundary is based on two fundamental observables of the basic variables, the Holonomy-Flux algebra generated by:

Holonomies, $h_e(A) := \mathcal{P} \exp(\int_e A)$ **and**

Electric Fluxes, $E(f,S) := \int_S \mathrm{d}S^{ab} E^i_{ab}\, f^i$

(Ashtekar, AC, Zapata, CQG 15, 2955, 1998)

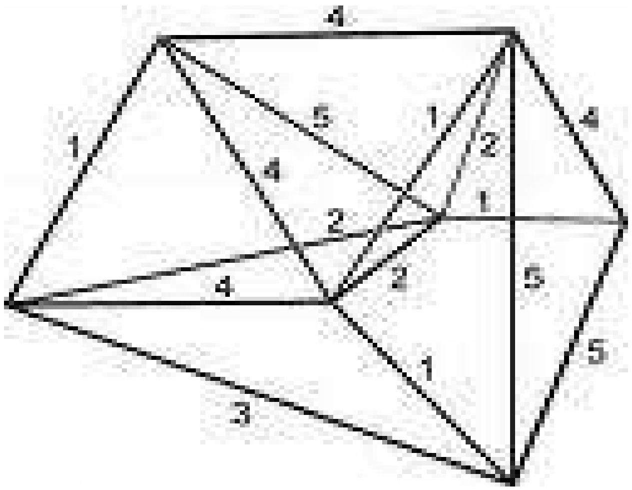
In electromagnetism (U(1) case):

$$h_\alpha(A) = \exp(i \oint_\alpha A \cdot \mathrm{d}\alpha) = \exp(i \int_S B \cdot \mathrm{d}S) \qquad \text{with} \quad \alpha = \partial S.$$

The main assumption of Loop Quantum Gravity is that these quantities become well defined operators. (LOST Theorem: There is a unique representation on a Hilbert space of these observables that is *diffeomorphism invariant*).

Hilbert space:

$$\mathcal{H}_{\rm AL} = \oplus_{\rm graphs}\, \mathcal{H}_{\Upsilon} = \text{Span of all Spin Networks } |\Upsilon, \vec{j}, \vec{m}\rangle \tag{1}$$

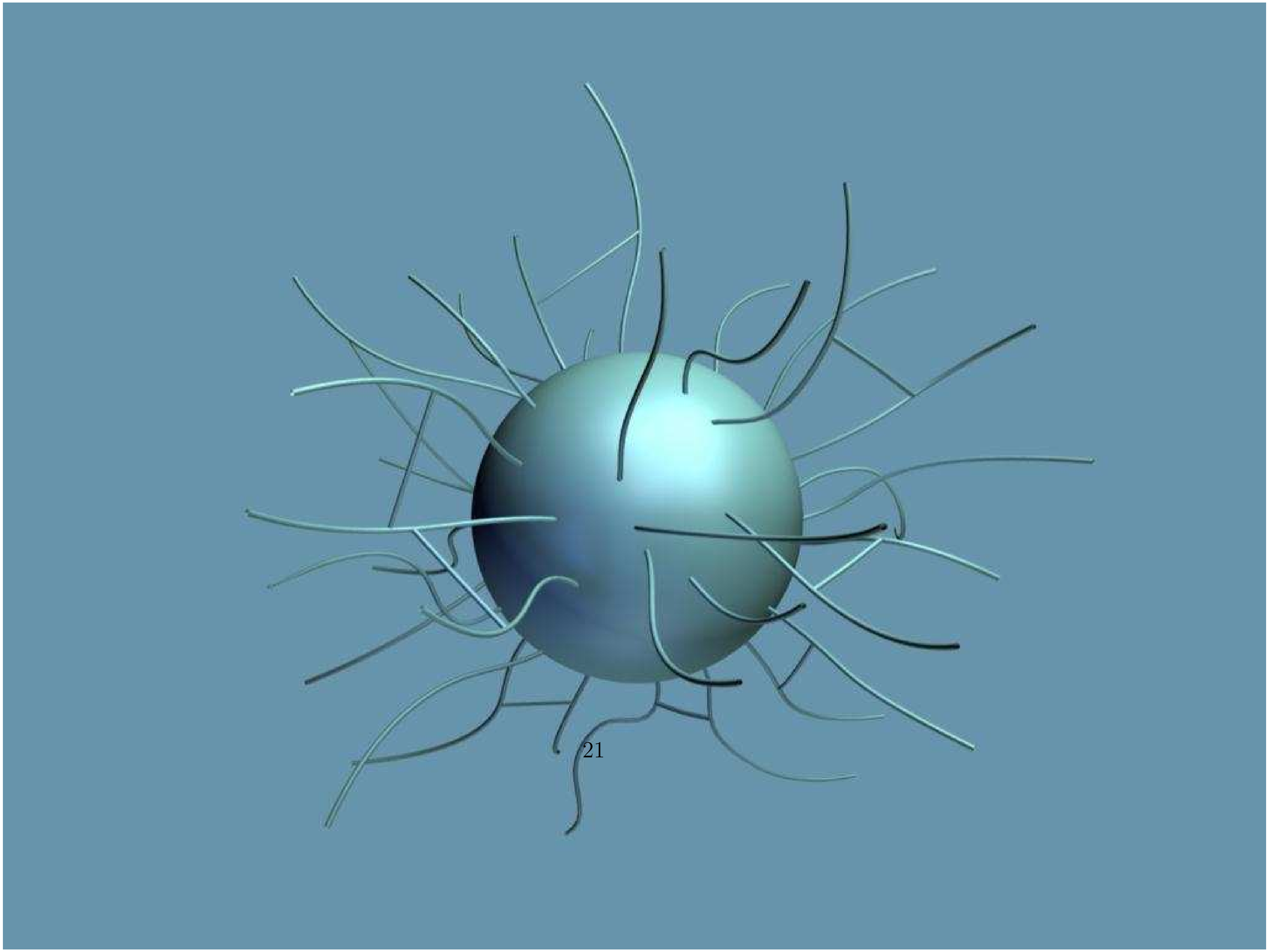


A Spin Network $|\Upsilon, \vec{j}, \vec{m}\rangle$ is a state labeled by a graph Υ, and some colorings $(\vec{j}, \vec{m})$ associated to edges and vertices.

The spin networks have a very convenient interpretation. They are the eigenstates of the quantized geometry, such as the area operator,

$$\hat{A}[S] \cdot |\Upsilon, \vec{j}, \vec{m}\rangle = 8\pi\ell_{\rm Pl}^2\gamma \sum_{\rm edges} \sqrt{j_i(j_i+2)}\; |\Upsilon, \vec{j}, \vec{m}\rangle \tag{2}$$

One sees that the edges of the graph, excite the quantum geometry of the surface S at the intersection points between S and Υ.



Quantum Horizons. (Ashtekar, Baez, AC, Krasnov, PRL 80 904, 1998)

- The boundary conditions are such that they capture the intuitive description of a horizon in equilibrium and allow for a consistent variational principle.

- One can use loop quantum geometry in the bulk and include the boundary.

- The quantum geometry of the horizon has independent degrees of freedom that fluctuate 'in tandem' with the bulk quantum geometry.

- The quantum boundary degrees of freedom are then responsible for the entropy.

- The entropy thus found can be interpreted as the entropy assigned by an 'outside observer' to the (2-dim) horizon $S = \Sigma \cap \Delta$.

HORIZON QUANTUM THEORY

Total Hilbert Space is of the form:

$$\mathcal{H} = \mathcal{H}_{\rm V} \otimes \mathcal{H}_{\rm S}$$

where $\mathcal{H}_{\rm S}$, the surface Hilbert Space, can be built from Chern Simons Hilbert spaces for a sphere with punctures.

The conditions on $\mathcal{H}$ that we need to impose are: Invariance under diffeomorphisms of S and the quantum condition on Ψ:

$$\left(\mathrm{Id} \otimes \hat{F}_{ab} + \frac{2\pi\,\gamma}{a_0}\,\hat{E}^i_{ab}\,r_i \otimes \mathrm{Id}\right) \cdot \Psi = 0$$

Then, the theory we are considering is a quantum gravity theory, with an isolated horizon of fixed area a_0 (and other multiple-moments). Physical state would be such that, in the bulk satisfy the ordinary constraints and, at the horizon, the quantum horizon condition.

ENTROPY

We are given a black hole of area a_0. What entropy can we assign to it? Let us take the microcanonical viewpoint. We shall count the number of horizon states $\mathcal{N}$ such that they are compatible with the macroscopic constraints and satisfy:

- **The area eigenvalue $\langle \hat{A} \rangle \in [a_0 - \delta, a_0 + \delta]$**
- **The quantum horizon condition.**

The entropy $\mathcal{S}$ will be then given by

$$\mathcal{S} = \ln \mathcal{N}.$$

The challenge now is to identify those states that satisfy the two conditions, and count them.

CHARACTERIZATION OF THE STATES

There is a convenient way of characterizing the states by means of the spin network basis. If an edge of a spin network with label j_i ends at the horizon S, it creates a puncture, with label j_i. The area of the horizon will be the area that the operator on the bulk assigns to it: $A = 8\pi\gamma\ell_{\rm Pl}^2 \sum_i \sqrt{j_i(j_i+1)}$.

Is there any other quantum number associated to the punctures p_i? Yes! the eigenstates of $\hat{E}_{ab}$ that are also half integers m_i, such that $-|j_i| \leq m_i \leq |j_i|$. The quantum horizon condition relates these eigenstates to those of the horizon Chern-Simons theory. The requirement that the horizon is a sphere (topological) then imposes a 'total projection condition' on $m's$:

$$\sum_i m_i = 0$$

A 'configuration' the quantum horizon is then characterized by a set of punctures p_i and to each one a pair of half integer (j_i, m_i).

The counting has three steps:

i) Given the classical area a_0, find the possible sets $\{n_k\}$ of configurations of j's compatible with it.

ii) Given such a configuration, $\{n_k\}$, find the degeneracy $R(\{n_k\})$ associated the possible orderings.

If we are given N punctures and two assignments of labels (j_i, m_i) and (j'_i, m'_i). Are they physically distinguishable? or a there some 'permutations' of the labels that give indistinguishable states?

That is, what is the statistics of the punctures?

As usual, we should let the theory tell us. One does not postu-

late any statistics. If one treats in a careful way the action of the diffeomorphisms on the punctures one learns that when one has a pair of punctures with the same labels j and m, then the punctures are indistinguishable and one should not count them twice. In all other cases the states are distinguishable.

iii) Given the degeneracy induced by the 'statistics', one has to find the degeneracy associated to the number of horizon states compatible with the configurations $\{n_k\}$. This step involves a choice. Are we going to keep track of both labels j_i, m_i? or are we just going to count horizon state, labelled by m's, that could come from some j's. This is the distinction between the DLM and GM countings. Since this is the step that knows about the horizon theory, it is at this point that a relation with CFT can be found.

THE COUNTING

We start with an isolated horizon, with an area a_0 and ask how many states are there compatible with the two conditions. Two relevant quantum numbers (j_I, m_I) for the Hilbert space.
Exact counting using number theory. Thus, given $(n_{1/2}, n_1, n_{3/2}, \ldots, n_{k/2})$, where $n_{s/2}$ is the number of punctures with label s we count the number of states:

$$\mathcal{N} = \sum_{\{n_s\}} \left(\frac{N!}{\Pi_s\,(n_s!)}\right) \; \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\theta \prod_s \cos(s\theta)^{n_s} \tag{3}$$

Taking the *large area approximation* $A \gg \ell_{\rm Pl}$**, and using the Stirling approximation. One gets:**

$$S = \frac{A}{4\ell_{\rm Pl}^2}\,\frac{\gamma_0}{\gamma} \tag{4}$$

with γ_0 the solution to $\sum_j 2\, e^{2\pi\,\gamma_0\sqrt{j_i(j_i+1)}} = 1$.

The first correction to the entropy area relation is

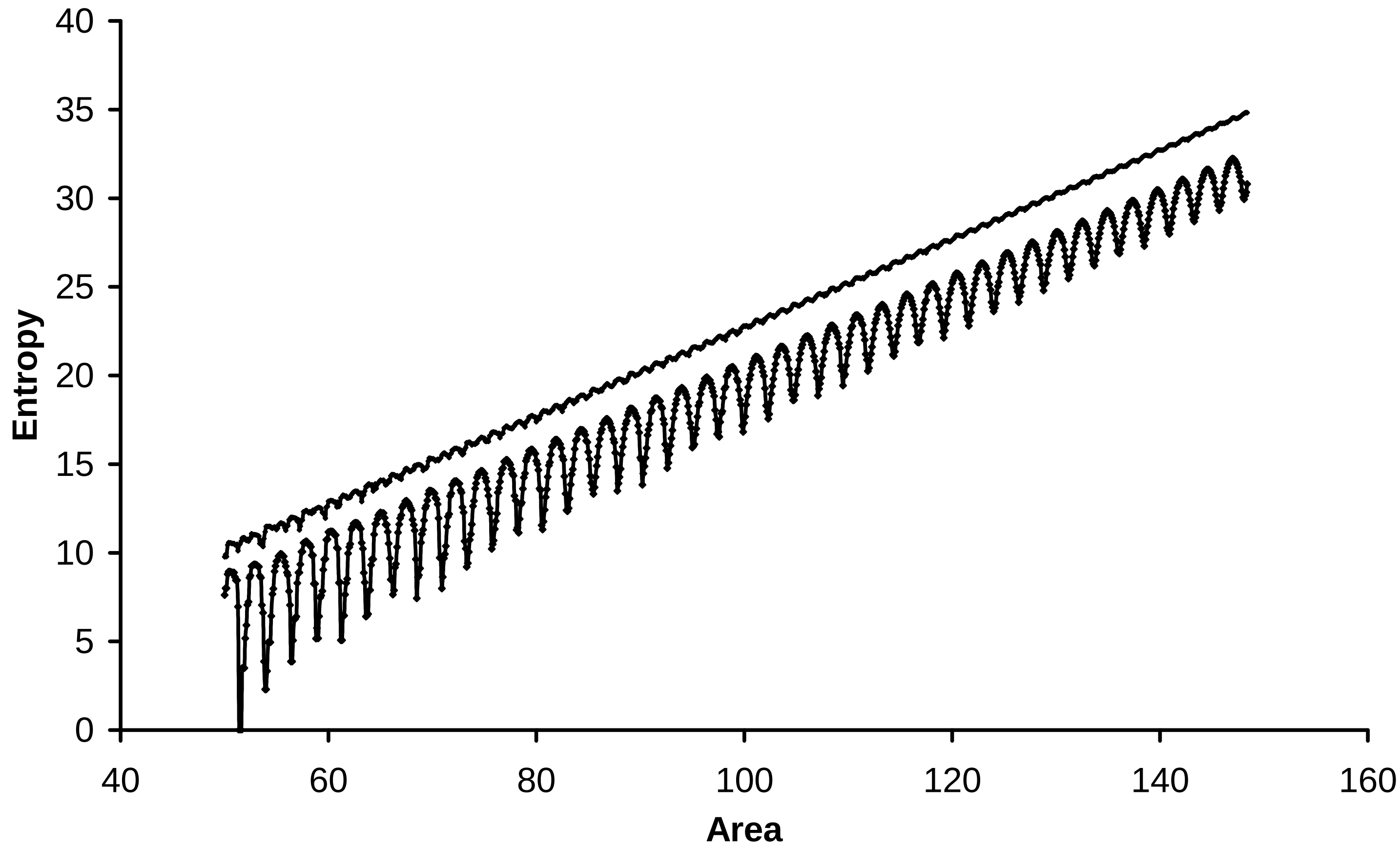
$$S = \frac{A}{4\ell_{\rm Pl}^2}\,\frac{\gamma_0}{\gamma} - \frac{1}{2}\ln(A) + \dots$$

- **If we want to make contact with the Bekenstein-Hawking we have to chose $\gamma = \gamma_0$.**
- **The coefficient of the logarithmic term is universal.**
- **The formalism can be generalized to more general situations, and the result is the same:**
  - **Maxwell, Dilatonic** (Ashtekar, AC, CQG 17 1317, 2000) **and Yang Mills Couplings** (Ashtekar, Krishnan, Fairhurst, PRD 2000)
  - **Cosmological, Distortion and Rotation** (Ashtekar, Engle, Van der Broeck, CQG 2004)
  - **Non-minimal Couplings.** (Ashtekar, AC, CQG 20 4473, 2003)

ENTROPY QUANTIZATION

When one performs an exact counting of states (both using a computer and using number theory methods) for small black holes, one finds new structures,

(AC, Díaz-Polo, Fernández-Borja, CQG 24 243 2007)


40
35
30
25
20
15
10
5
0
Entropy
40
60
80
100
120
140
160
Area

Both the oscillations found with a large value of δ as well as these structures in the 'spectrum' posses the same periodicity

$$\Delta A_0 \approx 2.41\ \ell_{\rm Pl}^2$$

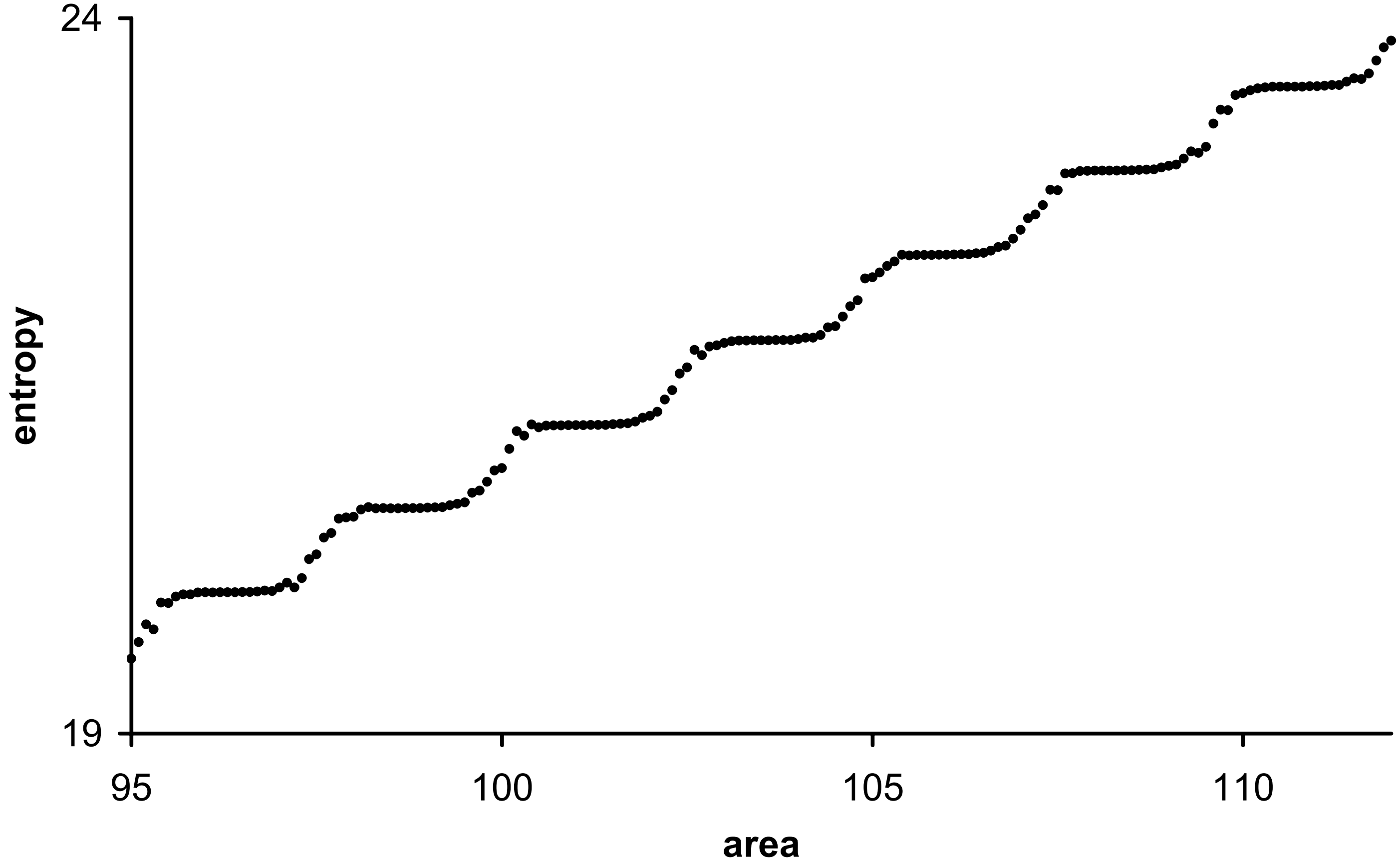
Is there any physical significance to this periodicity?

we chose the interval:

$$2\,\delta = \Delta A_0$$

With this choice, the plot of the entropy *vs* area becomes:

(AC, Díaz-Polo, Fernández-Borja, PRL 98, 181301 2007)


24
19
entropy
95
100
105
110
area

WHAT DOES THIS MEAN?

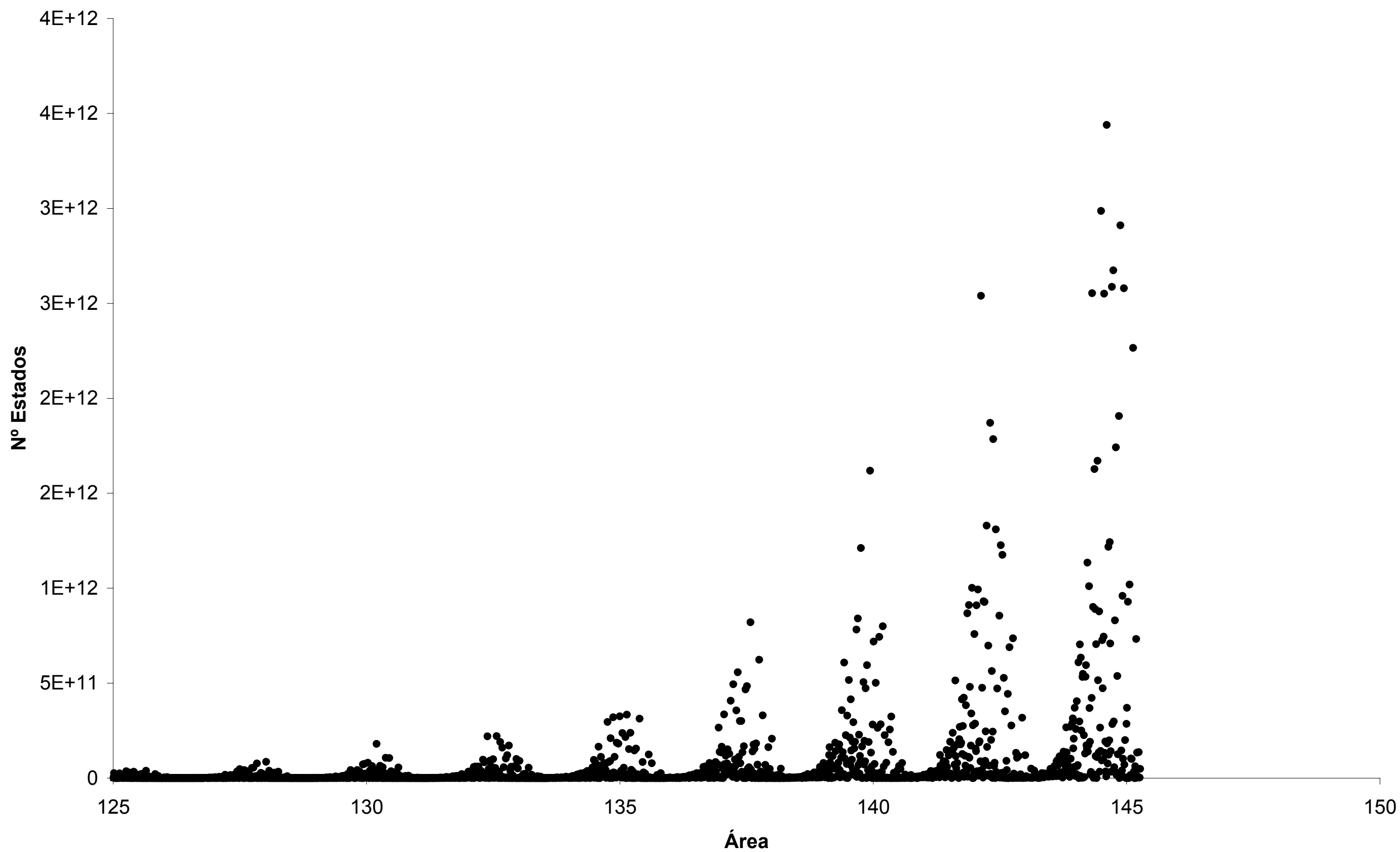
Instead of oscillations, Entropy seems to increase in discrete steps.

Furthermore, the height of the steps seems to approach a constant value as the area of the horizon grows, thus implementing in a rather subtle way the conjecture by Bekenstein that entropy should be equidistant for large black holes.

Is there any way of understanding this? Maybe

While the constant number in which the entropy of large black holes 'jumps' is:

$$\Delta S \mapsto 2\,\gamma_0\,\ln(3)$$



4E+12
4E+12
3E+12
3E+12
2E+12
2E+12
1E+12
5E+11
0
Nº Estados
125
130
135
140
145
150
Área

CONCLUSIONS AND TAKE HOME MESSAGE

- **Isolated Horizons provide a consistent framework to incorporate black holes locally in equilibrium.**

- **One can consistently quantize the theory.**

- **Entropy is** *finite* **and dominant term linear in Area.**

- **Any black hole of astrophysical interest is included**

- **Analysis of Planck scale BH's shows 'quantization of entropy'.**

- **Contact with Bekenstein's heuristic model, and Mukhanov-Bekenstein in a subtle manner**

OUTLOOK

- **We have not dealt with the singularity**
- **Ashtekar-Bojowald 'paradigm' for an extended quantum space-time**
- **Based on expectations about singularity resolution coming from LQC**
- **Hawking radiation?**
- **Lost Information Puzzle**
- **Full theory: How to specify quantum black holes from the full theory?**