Unruh effect in the General Boundary Formulation

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Unruh effect

The Unruh effect states that linearly uniformly accelerated observers perceive the Minkowski vacuum state (i.e. the no-particle state of inertial observers) as a mixed particle state described by a density matrix at temperature $T = \frac{a}{2\pi k_B}$, a being the constant acceleration of the observer.

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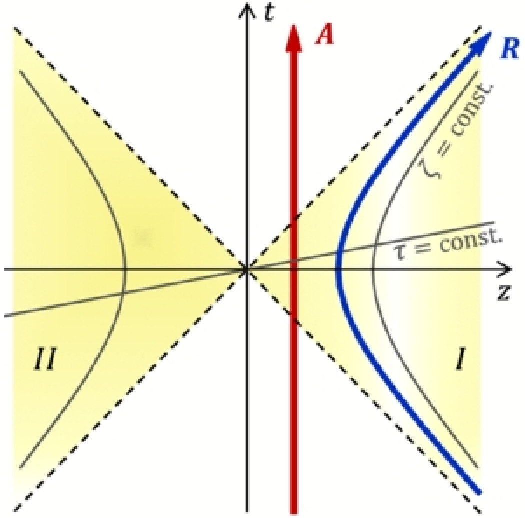
Importance:

- Relation between the Minkowski vacuum and the notion of particle in Rindler space (naturally associated with an accelerated observer): particle content of a field theory is observer dependent
- Relation with the Hawking effect and cosmological horizons
- Possible experimental detection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- **Operational interpretation**: A uniformly accelerated Unruh-DeWitt detector responds as if submersed in a thermal bath when interacting with a quantum field in the Minkowski vacuum state.

- **Operational interpretation**: A uniformly accelerated Unruh-DeWitt detector responds as if submersed in a thermal bath when interacting with a quantum field in the Minkowski vacuum state.

- **Particle interpretation**: the vacuum state in Minkowski corresponds to an entangled state between the modes of the field defined in the left and right Rindler wedges.
  - Crispino et al., *The Unruh effect and its applications*, Rev. Mod. Phys. **80** (2008), 787–838
    «the Unruh effect is the equivalence between the Minkowski vacuum and a thermal bath of Rindler particles»

Outline

General Boundary Formulation of Quantum Theory

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The GBF is a new axiomatic formulation of quantum theory based on

1. the mathematical framework of topological quantum field theory
2. A generalization of the Born's rule

- Motivated by the problem of quantum gravity
- Offers a new perspective on quantum theory
- Can treat situation where QFT fails, e.g. static black hole, AdS [see the talk of Max Dohse]
- May solve interpretation problems of background independent QFT (locality, problem of time)
- Is compatible with some approaches to quantum gravity (3d QG, spin foams, GFT)

Basic structures

In the GBF algebraic structures are associated to geometric ones.

Geometric structures (representing pieces of spacetime):

- **hypersurfaces**: oriented manifolds of dimension $d-1$
- **regions**: oriented manifolds of dimension d with boundary

Algebraic structures:

- To each hypersurface Σ associate a Hilbert space $\mathcal{H}_\Sigma$ of states.
- To each region M with boundary ∂M associate a linear amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$
- As in AQFT, observables are associated to spacetime regions: An observable O in a region M is a linear map $\rho_M^O : \mathcal{H}_{\partial M} \to \mathbb{C}$, called observable map.

Axioms and recovering of standard results

These algebraic structures are subject to a number of axioms, in the spirit of **TQFT**.

- Standard transition amplitudes of QFT can be recover from the GBF: $\rho_{[t_1,t_2]}(\psi_{t_1} \otimes \eta_{t_2}) = \langle \eta | U(t_1, t_2) | \psi \rangle$.
- A consistent probability interpretation can be implemented standard probabilities recovered.
- Conventional expectation values of observable can be recovered.

Outline

GBF and QFT

Standard QFT can be formulated within the GBF

GBF and QFT

Standard QFT can be formulated within the GBF

2 quantization schemes have been studied, that transform a classical field theory into a general boundary quantum field theory:

- Schrödinger-Feynman quantization
- holomorphic quantization

Schrödinger-Feynman quantization

- Schrödinger representation + Feynman path integral quantization
  The state space $\mathcal{H}_\Sigma$ for a hypersurface Σ is the space of functions on field configurations K_Σ on Σ.
- Inner product,

$$\langle\psi_2|\psi_1\rangle = \int_{K_\Sigma} \mathcal{D}\varphi\, \psi_1(\varphi)\overline{\psi_2(\varphi)}.$$

- Amplitude for a region M, $\psi \in \mathcal{H}_{\partial M}$,

$$\rho_M(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi\, \psi(\varphi) \int_{K_M, \phi|_{\partial M}=\varphi} \mathcal{D}\phi\, e^{\mathrm{i}S_M(\phi)}.$$

- A classical observable F in M is modeled as a function on K_M. The quantization of F is the linear map $\rho_M^F : \mathcal{H}_{\partial M} \to \mathbb{C}$ defined as

$$\rho_M^F(\psi) = \int_{K_{\partial M}} \mathcal{D}\varphi\, \psi(\varphi) \int_{K_M, \phi|_{\partial M}=\varphi} \mathcal{D}\phi\, F(\phi) e^{\mathrm{i}S_M(\phi)}.$$

Holomorphic quantization

- Linear field theory: L_Σ is the vector space of solutions near the hypersurface Σ.
- L_Σ carries a non-degenerate symplectic structure ω_Σ and a complex structure $J_\Sigma : L_\Sigma \to L_\Sigma$ compatible with the symplectic structure:

$$J_\Sigma^2 = -\mathrm{id}_\Sigma \qquad \text{and} \qquad \omega_\Sigma(J_\Sigma(\cdot), J_\Sigma(\cdot)) = \omega_\Sigma(\cdot,\cdot).$$

- J_Σ and ω_Σ combine to a real inner product $g_\Sigma(\cdot,\cdot) = 2\omega_\Sigma(\cdot, J_\Sigma \cdot)$ and to a complex inner product $\{\cdot,\cdot\}_\Sigma = g_\Sigma(\cdot,\cdot) + 2\mathrm{i}\omega_\Sigma(\cdot,\cdot)$ which makes L_Σ into a complex Hilbert space.
- The Hilbert space $\mathcal{H}_\Sigma$ associated with Σ is the space of holomorphic functions on L_Σ with the inner product

$$\langle\psi,\psi'\rangle_\Sigma = \int_{L_\Sigma} \overline{\psi(\phi)}\psi'(\phi) \exp\left(-\frac{1}{2} g_\Sigma(\phi,\phi)\right) \mathrm{d}\mu(\phi),$$

where μ is a (fictitious) translation-invariant measure on L_Σ.

Holomorphic quantization (II)

- The amplitude map $\rho_M : \mathcal{H}_{\partial M} \to \mathbb{C}$ associated with the spacetime region M for a state $\psi \in \mathcal{H}_{\partial M}$ is given by

$$\rho_M(\psi) = \int_{L_\Sigma} \psi(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

- The observable map associated to a classical observable F in a region M is

$$\rho_M^F(\psi) = \int_{L_\Sigma} \psi(\phi) F(\phi) \exp\left(-\frac{1}{4} g_{\partial M}(\phi, \phi)\right) \mathrm{d}\mu_{\tilde{M}}(\phi).$$

Result

The GBF axioms are satisfied by these quantization prescriptions.

Klein-Gordon theory in Minkowski

- Action of a real massive Klein-Gordon field on $1+1$-dimensional Minkowski spacetime

$$S[\phi] = \frac{1}{2}\int \mathrm{d}^2x\, \left(\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - m^2\phi^2\right).$$

- The GBF is defined in a region M bounded by the disjoint union of two spacelike hypersurfaces represented by two equal time hyperplanes.
- It is convenient to expand the field in the basis of the boost modes

$$\psi_p(x,t) = \frac{1}{2^{3/2}\pi}\int_{-\infty}^{\infty} \mathrm{d}q\, \exp\left(\mathrm{i}m(x\sinh q - t\cosh q) - \mathrm{i}pq\right)$$

Klein-Gordon theory in Minkowski

All the relevant structures can be defined and the Hilbert space constructed.

- The complex structure results to be

$$J_{\Sigma_i} = \frac{\partial_t}{\sqrt{-\partial_t^2}}$$

- The vacuum state is the standard Minkowski vacuum state
- Amplitude and observable maps are implementable in terms of $\omega(\cdot,\cdot), g(\cdot,\cdot)$ and $\{\cdot,\cdot\}$

Klein-Gordon theory in Rindler space

- Rindler space is defined by $ds^2 = \rho^2 d\eta^2 - d\rho^2$, where

$$t = \rho \sinh \eta, \qquad x = \rho \cosh \eta$$

It corresponds to the right wedge of Minkowski space, $\mathcal{R} := \{x \in \mathcal{M} : x^2 \leq 0,\ x > 0\}$.

- We consider the region $R \subset \mathcal{R}$ bounded by the disjoint union of two equal-Rindler-time hyperplanes.
- The field is expanded in the basis of the Fulling modes

$$\phi_p^R(\rho, \eta) = \frac{(\sinh(p\pi))^{1/2}}{\pi} K_{\mathrm{i}p}(m\rho) e^{-\mathrm{i}p\eta}, \qquad p > 0,$$

$K_{\mathrm{i}p}$ is the modified Bessel function of the second kind (Macdonald function).

Klein-Gordon theory in Rindler space

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- Amplitude and observable maps are implementable in terms of $\omega(\cdot,\cdot), g(\cdot,\cdot)$ and $\{\cdot,\cdot\}$

Boundary condition

In order for the quantum theory to be **well defined** the following condition must be imposed

$$\phi^R(\rho = 0, \eta) = 0$$

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The relevance of this boundary condition manifests at the level of the algebraic structures, e.g.

$$\omega^{(\mathcal{R})}_{\Sigma_0}(\phi, \phi') = \omega_{\Sigma_0^R}(\phi^R, \phi^{R\,\prime}) + \lim_{\epsilon \to 0} \mathrm{i} \int_0^{\epsilon} \mathrm{d}p \, \frac{\cosh(p\pi)}{\sinh(p\pi)} \left[\phi(p) \, \overline{\phi(p)'} - \overline{\phi(p)} \, \phi'(p) \right],$$

where Σ_0 hyperplane $t = 0$, Σ_0^R is the semi-hyperplane $\eta = 0$, $\Sigma_0^R = \Sigma_0 \cap \mathcal{R}$.

$\Rightarrow$ the two quantum theories, in Minkowski and in Rindler spaces, are inequivalent

Outline

Unruh effect

Two notions:

1. **Global Unruh effect**: Comparison of particle states in Minkowski and Rindler spaces, e.g.
   - Crispino et al.:
     «The Unruh effect is defined in this review as the fact that the usual vacuum state for QFT in Minkowski spacetime restricted to the right Rindler wegde is a thermal state.»
   - Jacobson:
     «The essence of the Unruh effect is the fact that the density matrix describing the Minkowski vacuum, traced over the states in the region $z < 0$, is precisely a Gibbs state for the boost Hamiltonian at a temperature $T = 1/2\pi$.»
2. **Local Unruh effect**: Comparison of expectation values of local observables, namely observable with compact support both in Minkowski and Rindler space.

Global Unruh effect

- Because of the inequivalence between the QFTs, no direct identification of Minkowski quantum states with Rindler quantum states is possible.
- **There is no global Unruh effect!**
- Same critique of the Russian school of Belinskii et al.

Local Unruh effect

- We consider the Weyl observable

$$F(\phi) = \exp\left(\mathrm{i}\int \mathrm{d}^2x\, \mu(x)\phi(x)\right),$$

$\mu(x)$ has compact support in the interior of the right wedge $\mathcal{R}$. F is a well defined observable in both Minkowski and Rindler spaces.

- We compute the expectation value of F
  1. on the Minkowski vacuum state

$$K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}},$$

  where K_{0,Σ_i} is the Minkowski vacuum state in $\mathcal{H}_{\Sigma_i}$, $(i = 1, 2)$, and
  2. on the Rindler mixed state

$$D = \prod_i (1 - \exp(-2\pi k_i)) \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!(2k_i)^{n_i}} \psi_{n_i} \otimes \overline{\psi_{n_i}},$$

  ψ_{n_i} is the Rindler state with n_i particles defined in $\mathcal{H}_{\Sigma_i^R}$, $(i = 1, 2)$.

Local Unruh effect

Using the observable map we compute the two expectation values:

- Expectation value in Minkowski space

$$\rho_M^F(K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}}) = \exp\left(\frac{\mathrm{i}}{2}\int \mathrm{d}^2x\,\mathrm{d}^2x'\;\mu(x)G_F^{\mathcal{M}}(x,x')\mu(x')\right),$$

  where $G_F^{\mathcal{M}}(x,x')$ is the Feynman propagator in Minkowski.

Local Unruh effect

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  where $G_F^{\mathcal{M}}(x,x')$ is the Feynman propagator in Minkowski.

- Expectation value in Rindler space

$$\rho_R^F(D) = \prod_i N_i^2 \sum_{n_i=0}^{\infty} \frac{e^{-2\pi n_i k_i}}{(n_i)!(2k_i)^{n_i}}\, N^{-2}\int \mathrm{d}\xi_1\,\mathrm{d}\overline{\xi_1}\,\mathrm{d}\xi_2\,\mathrm{d}\overline{\xi_2}\;\rho_R^F(K_{\xi_1}\otimes\overline{K_{\xi_2}})$$
$$\exp\left(-\frac{1}{2}\int\frac{\mathrm{d}k}{2k}|\xi_1(k)|^2\right)(\xi_1(k_i))^{n_i}\;\exp\left(-\frac{1}{2}\int\frac{\mathrm{d}k}{2k}|\xi_2(k)|^2\right)(\overline{\xi_2(k_i)})^{n_i},$$

  where the n-particle states have been expanded in the basis of the coherent states K_{ξ_i}

Local Unruh effect

The result of the computation is

$$\boxed{\rho^F_M(K_{0,\Sigma_1} \otimes \overline{K_{0,\Sigma_2}}) = \rho^F_R(D)}$$

The local Unruh effect exists!

Conclusions and outlook

Conclusions

- Successfull implementation of the GBF in Rindler space
- New perspective on the Unruh effect: the distinction of the notions of global and local Unruh effect offers a clarification between different positions on the Unruh effect.
- First application of the amplitude map and implementation of the Berezin-Toeplitz quantization scheme.

Outlook

- Construction of the GBF for more general spacetime regions (in particular regions avoiding the origin of Minkowski spacetime) [work in progress]
- Composition of hypersurfaces and corresponding algebraic structures
- Relation with the Hawking effect