Prospectos en Topología
Semester 2025-2

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During the 2025-2 term, the Seminar "Prospectos en Topología" will have the following two thematic blocks:

1. Colorful applications of the Davis–Januszkiewicz space and unstable vector bundles.
2. Classifying spaces for commutativity of geometric orientable 3-manifold groups.

The seminar will also feature a number of individual talks. The schedule for the seminar is as follows:

- Individual Talk: "A descriptive set-theoretic approach to manifold classification"
  - **Speaker:** Jeffrey Bergfalk.
  - **Date:** January 27th.
  - **Abstract:** In this work, joint with Iian Smythe, we describe a unified descriptive set theoretic framework for the study and comparison of classification problems for various classes of manifolds. Within this framework, we record several fundamental results, on the Borel complexity of the homeomorphism problem for compact manifolds or 2-manifolds, for example, and of the isometry problem for hyperbolic manifolds, and for algebraically finite hyperbolic manifolds of low dimension. We will close with a list of some of the most conspicuous open questions.
- Individual Talk: "On Vector Bundles and K-theory of Certain Orbifold Toroidal Quotients"
  - **Speaker:** Mario Andrés Velásquez Méndez.
  - **Date:** February 10th.
  - **Abstract:** Let m be a square-free positive integer, and let A be an $n \times n$ integer matrix of order m. The matrix A defines an action of the cyclic group of order m on the n-dimensional torus. In this talk we describe a method for computing the topological K-theory of the resulting toroidal orbifold for any matrix A. This is achieved by considering the associated semidirect product of a free abelian group of rank n with a cyclic group of order m, where the conjugacy action is defined using the matrix A and its classifying space for proper actions. This work is a collaboration with Luis Jorge Sánchez.
- Individual Talk: "Descriptive Set Theory and Geometric Group Theory"
  - **Speaker:** Forte Shinko.
  - **Date:** February 17th.
- Colorful applications of the Davis–Januszkiewicz space and unstable vector bundles

  Polyhedral products are part of Toric Topology, a really active branch of mathematics in which Algebraic Topology, Algebraic Geometry and Combinatorics interact. The objective of these series of talks is to focus on an application of the Davis–Januszkiewicz space and unstable complex vector bundles to colorings.

Given a simplicial complex K, a map $\lambda\colon V(K) \longrightarrow \{1,\ldots,m\}$ is a $(\underline{m}, s)$-coloring if there are no monochromatic simplices of size larger than s. If $s = 1$ we say that it is a proper coloring. In [DJ91] for a polytope the Davis-Januszkiewicz space was defined and the authors noticed that if a certain bundle splits into a direct sum of n complex line bundles and a trivial bundle, the simplicial complex dual to the boundary of a polytope of dimension n admits a proper coloring with n colors. This result was generalized in [Not10] to any simplicial complex and in [DMN] this was extended to other values of s.

This thematic block will consist of three talks. The first two will be a general introduction to the unstable classification of vector bundles and the third one will be focused on colorful applications of vector bundles over the Davis-Januszkiewicz space.

1. General introduction to the unstable classification of complex vector bundles
   - **Speaker:** Noé Bárcenas Torres.
   - **Date:** March 3rd and March 10th.
   - **Abstract:** While the *stable* classification of vector bundles goes through the definition of complex K-theory as a cohomology theory, the *unstable* classification of complex or real vector bundles fundamentally uses Obstruction Theory, Homotopy Theory and Functor Calculus to understand the set of isomorphism classes of complex vector bundles over a CW-complex prior to group completion. In these talks we will compare the stable and unstable settings.
   - **References:** [AR76], [Opi24] and [Swi79].
2. Colorful applications of vector bundles over the Davis-Januszkiewicz space

   Speaker: Andrés Carnero Bravo.
   - **Date:** March 31th.
   - **References:** [Ero14], [Hua20] and [NN05].

- Individual Talk: "Twisted geometric K-homology"
  - **Speaker:** Michael Joachim.
  - **Date:** March 24th.
- Individual Talk: "The weak Ramsey property and extreme amenability"
  - **Speaker:** Adam Bartoš.
  - **Date:** April 7th.
  - **Abstract:** We extend the Kechris-Pestov-Todorčević correspondence to weak Fraïssé categories and automorphism groups of generic objects. The new ingredient is the weak Ramsey property. We demonstrate the theory on several examples including monoid categories, the category of almost linear orders, and categories of strong embeddings of trees.
  - **References:** [BBDBK24].
- The classifying space for commutativity of geometric orientable 3-manifold groups

For a topological group G let $E_{\text{com}}(G)$ be the total space of the universal transitionally commutative principal G-bundle as defined by Adem-Cohen-Torres-Giese. This space is often referred to as the *classifying space for commutativity* of G.

For a discrete group G the space $E_{\text{com}}(G)$ is homotopy equivalent to the geometric realization of the order complex of the poset of cosets of abelian subgroups of G. In these series of talks we discuss classifying spaces for commutativity of fundamental groups of closed orientable geometric 3-manifolds.

  - **Speaker:** Luis Eduardo García Hernández.

- **Dates:** April 21st and April 28th.
- **Contents:**
  1. Algebraic Topology Tools:
     * Fundamental Group.
     * Simplicial Sets and Simplicial Complexes.
     * Homotopic Constructions and Quillen's Theorem A.
  2. Groups and Associated Partial Orders:
     * Finite Groups and their Presentations.
     * POSET of Abelian Subgroups.
     * Order Complex of a Group.
  3. The Classifying Space for Commutativity of a Group
     * Original Definition.
     * Alternative Models of E_{com} for Discrete Groups.
  4. Fundamental Groups of 3-manifolds:
     * Geometries.
     * Geometric 3-manifolds.
     * JSJ Decompositions.
- **References:** [Thu97] and [ACGHS23].

• Individual Talk:

- **Speaker:** Federico Vigolo.
- **Date:** May 5th.

• Individual Talk: "s-cobordism and h-cobordism"

- **Speaker:** Oscar Romero.
- **Date:** May 12th.
- **Abstract:** In this talk, we will discuss the s-cobordism Theorem due to D. Barden, B. Mazur and J. Stallings. We will also see how the s-cobordism Theorem implies the h-cobordism Theorem for dimensions greather than or equal to 6 and the generalized Poincaré conjecture, i.e. for dimensions greather than or equal to 5.
- **Reference:** [KL05, Chapters 7-8].

• Individual Talk:

- **Speaker:** Sahana Balasubramanya.
- **Date:** May 19th.

• Individual Talk: "Isometric Rigidity of Metric Constructions with respect to Wasserstein Spaces"

- **Speaker:** Mauricio Che.
- **Date:** May 26th.
- **Abstract:** In this talk we focus on the isometric rigidity of certain classes of metric spaces with respect to the p-Wasserstein space. We prove that spaces that split a separable Hilbert space are not isometrically rigid with respect to $\mathbb{P}_2$. We then prove that infinite rays are isometrically rigid with respect to $\mathbb{P}_p$ for any $p \geq 1$, whereas taking infinite half-cylinders (i.e. product spaces of the form $X \times [0, \infty)$) over compact non-branching geodesic spaces preserves isometric rigidity with respect to $\mathbb{P}_p$, for $p > 1$. Finally, we prove that spherical suspensions over compact spaces with diameter less than $\pi/2$ are isometrically rigid with respect to $\mathbb{P}_p$, for $p > 1$.
- **References:** [CGGKSR24].

References

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