Prospectos en Topología
Semester 2025-1

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During the 2025-1 term, the Seminar "Prospectos en Topología" will have the following two thematic blocks:

1. Asymptotic dimension, property A and coarse embeddings into Hilbert Spaces.
2. Polyhedral products.

The seminar will also feature a number of individual talks. The schedule for the thematic blocks is as follows:

- Asymptotic dimension, property A and coarse embeddings into Hilbert Spaces

  Objective: To give a brief introduction to the concepts of (finite) asymptotic dimension, property A and coarse embeddability into Hilbert spaces for metric spaces of bounded geometry. We will also see how to make sense of these concepts for topological groups. As an example, we will compute the (infinite) asymptotic dimension of several big mapping class groups.

  1. Asymptotic dimension, property A and coarse embeddability into Hilbert spaces for metric spaces of bounded geometry
     - **Speaker:** Noé Bárcenas Torres.
     - **Date:** August 12th.
     - **References:** [BF08], [Yu00] and [Now07]
  2. Asymptotic dimension, property A and coarse embeddability into Hilbert spaces for coarse spaces and topological groups
     - **Speaker:** Carlos Pérez Estrada.
     - **Date:** August 19th and August 26th.
     - **Abstract:** In these two sessions we first review the notion of coarse space and how to generalize the concepts of bounded geometry, asymptotic dimension, property A and coarse embeddability into Hilbert spaces for coarse spaces. Then we will see how to naturally consider a topological group as a coarse space following Christian Rosendal, and how we could study property A and coarse embeddability of Polish groups into Hilbert spaces following the work of Kang Li for locally compact groups.
     - **References:** [Roe03], [Ros22] and [Li15].
  3. Asymptotic dimension of big mapping class groups
     - **Speaker:** Carlos Pérez Estrada.
     - **Date:** September 2nd.
     - **Abstract:** In this session we review the work of Curtis Grant, Kasra Rafi and Yvon Verberne about the infinite asymptotic dimension of several big mapping class groups.
     - **References:** [GRV21],

- Individual Talk:
  - **Speaker:** Roberta Shapiro.

– **Date:** September 23rd.

- Polyhedral products

  Objective: To give an introduction to polyhedral products, some of their properties as some of their applications. Broadly speaking, given a simplicial complex Ξ, a product of topological spaces –one of the following: cartesian product, smash product or join– and a family of pairs of CW-complexes $(\underline{X}, \underline{A})$, the corresponding polyhedral Ξ-product is the union of the spaces $\Xi_{i\in\sigma}Y_i$, where σ is a simplex of K and

  $$Y_i = \begin{cases} X_i & \text{if } i \in \sigma \\ A_i & \text{if } i \notin \sigma \end{cases}$$

  These spaces are widely studied. We won't pretend to give the state of the art but concentrate in some result about their homotopy type as well as some of their invariants.

  1. Introduction to polyhedral products
     – **Speaker:** Andrés Carnero Bravo.
     – **Date:** September 30th.
     – **Abstract:** We will review the definition and basic properties of polyhedral products and its variants (smash, join).
     – **References:** [BBC20, BBCG10].
  2. Moment angle complexes and Davis-Januskievich spaces
     – **Speaker:** Andrés Carnero Bravo.
     – **Date:** October 7th.
     – **Abstract:** In this session we will focus in two particular polyhedral products, the Moment angle complex and the Davis-Januskievich spaces, where the Davis-Januskievich space has the homotopy type of the Borel construction for the moment angle complex for a torus action. After reviewing some properties of their properties we will be interested in the KO-theory for the Davis-Januskievich space.
     – **References:** [ABB$^+$14, DJ91].
  3. The Steenrod algebra of polyhedral products
     – **Speaker:** Andrés Carnero Bravo.
     – **Date:** October 14th.
     – **Abstract:** We will review the paper *grbric* about the mod 2 Steenrod algebra for the moment angle complex.
     – **References:** [AGI$^+$].

- Individual Talk:
  – **Speaker:** Rodrigo De Pool.
  – **Date:** October 28th.

- Individual Talk: "Condensed Mathematics, Whitehead's problem, extremally disconnected spaces and forcing"
  – **Speaker:** Jeffrey Bergfalk.
  – **Date:** November 4th.
  – **References:** [BLHv24].

- Individual Talk:
  – **Speaker:** Ahtziri González.
  – **Date:** November 11th.

References

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[Yu00] Guoliang Yu. The coarse Baum-Connes conjecture for spaces which admit a uniform embedding into Hilbert space. *Invent. Math.*, 139(1):201–240, 2000.